# Discounted Cash Flow
# Summary

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## Basic Elements

Fundamental Terms

Cash flow, taxes,
time, cost of capital
point-in-time, risk

Mathematical Tools

**Conditional Expectation** $\mathrm{E}_t[\cdot]$

5 rules:
Classical expectation
Linearity
Certain amount
Iterated expectation
Taking out what is known

subjective probability — "other probability"

Precise foundation

**Cost of Capital**

Definition:
$\tilde{k}_t =_{\mathrm{Def}} \frac{\mathrm{E}_t[\widetilde{CF}_{t+1}+\widetilde{V}_{t+1}]}{\widetilde{V}_t} - 1$
Assumption: $\tilde{k}_t$ deterministic

**Valuation Equation**

$$\widetilde{V}_t = \sum_{s=t+1}^{\infty} \frac{E_t[\widetilde{CF}_s]}{(1+k)^{\,s-t}}$$

No arbitrage, asset
can be duplicated

**Fundamental Theorem**

"risk-neutral" probability $Q$ with
$r_f = E_t^Q\left[\frac{\widetilde{CF}_{t+1}+\widetilde{V}_{t+1}}{\widetilde{V}_t} - 1\right]$

## Private Income Tax

dividend $\tau^D$, interest $\tau^I$

**Fundamental Theorem**

Now instead of $r_f$
$\widetilde{V}_t = \frac{E_t^Q[\widetilde{CF}_{t+1}+\widetilde{V}_{t+1}]}{1+r_f(1-\tau^I)}$

**Leverage anew**

retention $\Longrightarrow$ tax shield

**Valuation Equation and Retention**

$$\widetilde{V}_t^l = \widetilde{V}_t^u + \sum_{s=t+1}^{T} \frac{\tau^I\left(1-\tau^D\right) r_f \mathrm{E}_t^Q\left[\widetilde{A}_s\right]}{\left(1+r_f\left(1-\tau^I\right)\right)^{s-t}}$$

**Varying tax rate**

**Never use** $k^{\text{post tax}} = k\cdot(1-\tau)$

## Corporate Income Tax

Debt$^l$ and Equity$^u$

debt $\Longrightarrow$ tax shield

unlevering/relevering

Unlevered Firms:

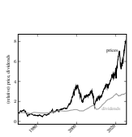
**Martingale-like cash flows**

we motivated why
$\mathrm{E}_t[\widetilde{CF}^u_{t+1}] = (1+g)\widetilde{CF}^u_t$

Conclusions:

**Gordon-Shapiro**
**CoC = Discount rates**

$\widetilde{V}_t^u = \frac{1+g}{k^{E,u}-g}\widetilde{CF}^u_t$
$\frac{\mathrm{E}_t^Q\left[\widetilde{CF}^u_{t+1}\right]}{1+r_f} = \frac{\mathrm{E}_t\left[\widetilde{CF}^u_{t+1}\right]}{1+k^{E,u}}$




Excursus: Excess volatility
(heroic assumption!)

Excursus:
**excess volatility**

Levered Firms

**Financing policy**

Valuation of tax shield
$\widetilde{V}_t^l = \widetilde{V}_t^u + \sum_{s=t+1}^{T} \frac{\tau r_f \mathrm{E}_t^Q\left[\widetilde{D}_s\right]}{(1+r_f)^{s-t}}$

Insolvency
two triggers

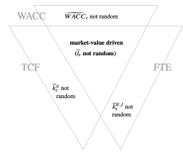
same Valuation of tax shield!
Over-indebtedness $\Longrightarrow$ Illiquidity

autonomous financing

deterministic debt $D$
APV formula
MoMi theorem
$\widetilde{V}_t^l = \widetilde{V}_t^u + \tau D$

based on market values

deterministic debt ratio $l$
three procedures




Relation $k^{E,u}$ and $WACC$?

**Adjustment Formulas**

Miles–Ezzell $1+\mathrm{WACC}_t = \left(1+k_t^{E,u}\right)\left(1-\frac{\tau r_f}{1+r_f}\,l_t\right)$
do not use MoMi adjustment