DCF: Personal Income Tax
Cost of Capital and Tax Rate

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The problem

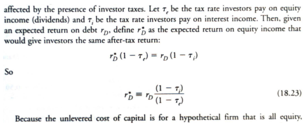
Up to now we have looked for the firm's value given the tax rate.

Now we ask for a varying tax rate τ.

This boils down to the question of how cost of capital $k^{E,\mathrm{u}}$ changes with τ.

(Remember: $k^{E,\mathbf{u}}$ **is post-tax!**)

Usually stated (Johansson 1969) 3



affected by the presence of investor taxes. Let τ_e be the tax rate investors pay on equity income (dividends) and τ_i be the tax rate investors pay on interest income. Then, given an expected return on debt r_D, define r_D^* as the expected return on equity income that would give investors the same after-tax return:

$$r_D^*(1-\tau_e) = r_D(1-\tau_i)$$

So

$$r_D^* \equiv r_D \frac{(1-\tau_i)}{(1-\tau_e)} \qquad (18.23)$$

Because the unlevered cost of capital is for a hypothetical firm that is all equity,

Berk/DeMarzo: Corporate Finance, 2007

The literature on valuation suggests a relation between cost of equity post-tax $k^{E,\mathrm{u}}$ and tax rate τ where

$$k^{E,\mathrm{u}} = k^E(1-\tau). \qquad (*)$$

k^E is sometimes interpreted as ‘cost of capital before–tax’.

Important is only the linearity: For example, increasing the tax rate from 0% to 50% lowers cost of capital by one half.

Nevertheless, **this equation is very problematic**.

Example

Look at a company that

- lives infinitely,
- has constant expected cash flows,
- no retainments and no investments.

For such a firm

$$\widetilde{CF}_t^{\mathrm{u}} = \widetilde{GCF}_t(1-\tau) \qquad (**)$$

holds, which is very convenient.

(Assumptions has to be made about gross instead of free cash flows because the tax rate will change.)

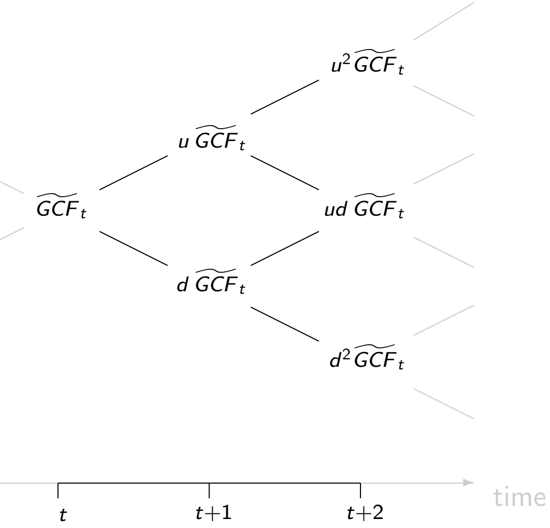
Then

$$\widetilde{V}_t = \frac{\widetilde{CF}_t^{\mathrm{u}}}{k^{E,\mathrm{u}}}$$

and from (*) with (**)

$$\widetilde{V}_t = \frac{\widetilde{CF}_t^{\mathrm{u}}}{k^{E,\mathrm{u}}} = \frac{\widetilde{GCF}_t(1-\tau)}{k^E(1-\tau)} = \frac{\widetilde{GCF}_t}{k^E}. \quad (***)$$

The personal income tax rate cancels! Personal taxes do not seem to have an influence on company value.



Consider our infinite example with gross cash flows following up with subjective probability $P(u)$, down with $P(d)$.

Free cash flows martingale-like

Gross cash flows (before tax!) are martingale-like.
Are free cash flows (post tax!) martingale-like as well?

$$
\begin{aligned}
\mathrm{E}_t\left[\widetilde{CF}^{\mathrm{u}}_{t+1}\right] &= \mathrm{E}_t\left[(1-\tau)\widetilde{GCF}_{t+1}\right] \\
&= (1-\tau)\,\mathrm{E}_t\left[\widetilde{GCF}_{t+1}\right] \\
&= (1-\tau)P(u)u\widetilde{GCF}_t + (1-\tau)P(d)d\widetilde{GCF}_t \\
&= \Big(\underbrace{P(u)u + P(d)d}_{:=1+g}\Big)(1-\tau)\widetilde{GCF}_t \\
&= (1+g)\widetilde{CF}^{\mathrm{u}}_t.
\end{aligned}
$$

Yes!

The market

Now consider two firms

	firm A	firm A'
up and down factor	u, d	u', d'
gross cash flows	$\widetilde{GCF}_t$	$\widetilde{GCF}'_t$
firm values	$\widetilde{V}_t$	$\widetilde{V}'_t$
cost of capital	k	k'
growth rate	$g \stackrel{!}{=} 0$	$g' \stackrel{!}{=} 0$

The up- and down-movements of both cash flows are perfectly correlated with probability $P(u)$ and $P(d)$.

Duplication

One risk-free bond with value B_t at time t. Risk-free interest rate after tax $r_f(1-\tau)$. We now duplicate payments of firm A' by portfolio of A and bond.

Portfolio contains

$n_B :=$ bonds and

$n_A :=$ shares of firm A

such that payments equal the dividend of A'. Or,

$$\overbrace{n_B B_t\left(1+r_f(1-\tau)\right)+n_A\left(\widetilde{GCF}_{t+1}(1-\tau)+\widetilde{V}_{t+1}\right)}^{\text{portfolio A and bond}} = \underbrace{\widetilde{GCF}'_{t+1}(1-\tau)+\widetilde{V}'_{t+1}}_{\text{A'}}.$$

To determine n_A and n_B we use (***) and this gives

$$n_B B_t \left(1 + r_f(1-\tau)\right) + n_A \left(1 + k_{t+1}(1-\tau)\right) \widetilde{V}_{t+1} = \left(1 + k'_{t+1}(1-\tau)\right) \widetilde{V}'_{t+1}$$

or, given the stochastic structure,

$$n_B \left(1 + r_f(1-\tau)\right) B_t + n_A \left(1 + k(1-\tau)\right) u \widetilde{V}_t = \left(1 + k'(1-\tau)\right) u' \widetilde{V}'_t$$
$$n_B \left(1 + r_f(1-\tau)\right) B_t + n_A \left(1 + k(1-\tau)\right) d \widetilde{V}_t = \left(1 + k'(1-\tau)\right) d' \widetilde{V}'_t.$$

This is a 2×2-system that can be solved:

$$n_B = \frac{\widetilde{V}'_t}{B_t} \frac{(u - u')(1 + k'(1 - \tau))}{u(1 + r_f(1 - \tau))}$$
$$n_A = \frac{\widetilde{V}'_t}{\widetilde{V}_t} \frac{u'(1 + k'(1 - \tau))}{u(1 + k(1 - \tau))}.$$

(All variables are uncertain.)

Furthermore, since the market is free of arbitrage, we must have

$$n_B B_t + n_A \widetilde{V}_t = \widetilde{V}'_t.$$

There are now three equations. Plugging them all together results in

$$\frac{u - u'}{1 + r_f(1-\tau)} + \frac{u'}{1 + k(1-\tau)} = \frac{u}{1 + k'(1-\tau)} \qquad (A)$$

and this is a relation between

- the cost of capital k, k' and r_f before taxes,
- the tax rate τ, and
- the parameters u and u'.

Equation (A) is a no arbitrage-condition. If it is not satisfied there is an arbitrage opportunity in the market.

But: It is also a quadratic equation and such an equation has only two solutions. These are

$\tau = 100\,\%$ and

$\tau = 0\%$.

For any other tax rate there must be an arbitrage opportunity. This violates our basic principle of valuation.

Our result is in fact surprising. Is there any intuition for it?

Notice that cost of capital $k^{E,u}$ and company value $\widetilde{V}_t$ are related to each other (like "two sides of a coin"). By determining a relation between cost of capital and tax rate we implicitly determine a relation between value and tax rate.

But this relation is highly non-linear which is the reason for our arbitrage opportunity.

Never ever use

$$k^{E,\mathrm{u}} = k^{E}(1-\tau)$$

when the tax rate τ changes.