DCF: Firm income tax
Insolvency triggers

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Insolvency triggers

We now define two insolvency triggers—inability to pay and negative equity—and discuss how they are related.

To do so, we need to define the coupon rate c.

Coupon rate c

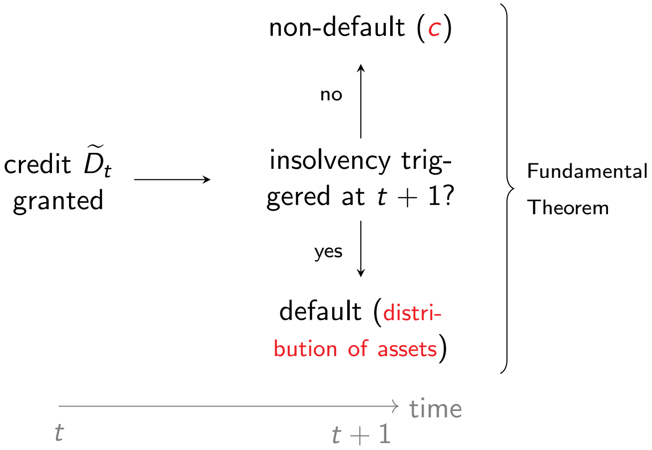


Figure: How is c determined?

Two insolvency triggers

We distinguish illiquidity and over-indebtedness. At time $t+1$ and state ω we have

illiquidity
: if cash flow is not sufficient to pay net debt payments, or

$$\widetilde{CF}^{\mathrm{l}}_{t+1}(\omega) < (1+c)\widetilde{D}_t(\omega) - \widetilde{D}_{t+1}(\omega)$$

over-indebtedness
: if firm value is less than debt value, or

$$\widetilde{V}^{\mathrm{l}}_{t+1}(\omega) < \widetilde{D}_{t+1}(\omega)$$

Actual law may differ. But within our framework, we adhere to the model and must formulate insolvency in its terms (point-in-time principle, prices are ex cash flow, information setup...)

Main result: Over-indebtedness implies illiquidity 5

Assume $c \geq r_f$.

Theorem: *If a levered company is over-indebted at time t in some state, then there is a (possibly later) date where the firm is illiquid.*

The converse (does illiquidity imply over-indebtedness?) is not necessarily true. But we return to this later...

By contradiction: consider an over-indebted firm that is never illiquid.

Since alway liquid

$$\widetilde{CF}^{\mathrm{l}}_s(\omega) \geq (1+c)\widetilde{D}_{s-1}(\omega) - \widetilde{D}_s(\omega).$$

We multiply with Q-probabilities (conditional at time t), use $c \geq r_f$ and add for all states and get

$$\mathrm{E}^Q_t[\widetilde{CF}^{\mathrm{l}}_s] \geq \mathrm{E}^Q_t\left[(1+r_f)\widetilde{D}_{s-1} - \widetilde{D}_s\right].$$

Dividing by $(1+r_f)^{s-t}$ and adding over all s

$$\widetilde{V}_t^{\mathrm{l}} = \sum_{s=t+1}^{T} \frac{\mathrm{E}_t^Q[\widetilde{CF}_s^{\mathrm{l}}]}{(1+r_f^{s-t})} \geq \sum_{s=t+1}^{T} \frac{\mathrm{E}_t^Q\left[(1+r_f)\widetilde{D}_{s-1} - \widetilde{D}_s\right]}{(1+r_f)^{s-t}}$$

$$= \sum_{s=t+1}^{T} \frac{\mathrm{E}_t^Q\left[\widetilde{I}_s + \widetilde{Pr}_s\right]}{(1+r_f)^{s-t}} \qquad \text{last lecture}$$

$$= \widetilde{D}_t$$

and the firm cannot be over-indebted. Contradiction.

Up to that point, default implications (partial write-off, parts of assets transferred to creditors; claims be deferred or forgiven. . .) were ignored.

First and rough approximation: upon default complete transfer of assets

default at ω in $t+1$

$$\Longrightarrow \qquad \underbrace{\widetilde{I}_{t+1}(\omega) + \widetilde{Pr}_{t+1}(\omega)}_{\text{repayment to creditors}} := \underbrace{\widetilde{CF}^{\mathrm{l}}_{t+1}(\omega) + \widetilde{V}^{\mathrm{l}}_{t+1}(\omega)}_{\text{all assets}}$$

This has profound implications.

Combine both insolvency triggers (a trigger is activated when it becomes negative):

$$\text{illiquidity} \qquad\qquad + \text{over-indebtedness}$$

$$= \widetilde{CF}^{l}_{t+1} + \widetilde{D}_{t+1} - (1+c)\widetilde{D}_{t} + \widetilde{V}^{l}_{t+1} - \widetilde{D}_{t+1}$$

$$= \underbrace{\widetilde{CF}^{l}_{t+1} + \widetilde{V}^{l}_{t+1}}_{\text{all assets}} - \underbrace{(1+c)\widetilde{D}_{t}}_{\text{creditor's claims at } t+1}$$

What happens if one trigger is met but the other is not?

Complete transfer of assets

no trigger met no insolvency, inequality irrelevant

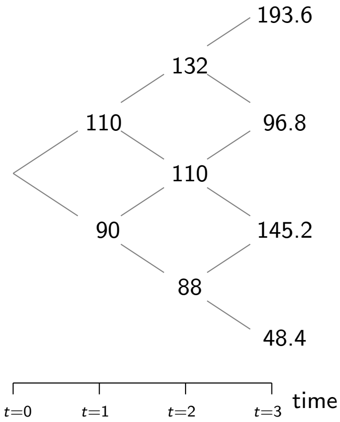
both trigger met sum is negative: creditor's claims are larger then all assets ("typical case")

one trigger met the sum can be positive: creditor's claims could be lower then all assets?!

We now present an example in which this last case occurs.

Finite example with insolvency

We start with our well-known example, have $k = 20\%$



levered cash flows $\widetilde{CF}^l$

and we add leverage

$$\color{red}{D_0 = 140,\ D_1 = 62},\ D_2 = 0.$$

At time $t = 2$ the company is unlevered.

For simplicity, we ignore taxes. They add complexity without adding insight.

From the lecture on "Cost of capital" we know the value at time $t = 0$,

$$\begin{aligned} V_0^l &= \frac{\mathrm{E}\left[\widetilde{CF}_1^{\,l}\right]}{1+k} + \frac{\mathrm{E}\left[\widetilde{CF}_2^{\,l}\right]}{(1+k)^2} + \frac{\mathrm{E}\left[\widetilde{CF}^{\,l}\right]}{(1+k)^3} \\ &= \frac{100}{1.2} + \frac{110}{1.2^2} + \frac{121}{1.2^3} \approx \underbrace{229.75}_{\text{firm value } V} > \underbrace{140}_{\text{debt value } D} \end{aligned}$$

Hence, the company is not over-indebted at $t = 0$.

Finite example with insolvency (cont.)

From the lecture on "Cost of capital" we know the value at time $t = 1$,

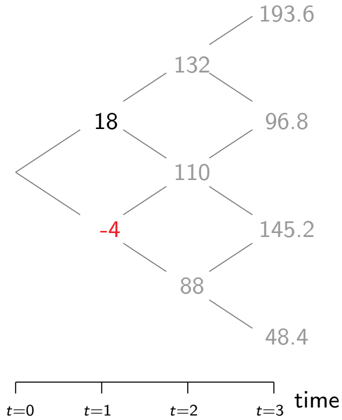
$$\widetilde{V}_1 \approx \begin{cases} 193.26, & \text{up} \\ 158.13, & \text{down} \end{cases} > 62 = D_1$$

Hence, the company is also not over-indebted at $t = 1$.

But what about illiquidity? We start with $c = r_f$ and evaluate the cash flows to owners, for example at $t = 1$ given by

$$\widetilde{CF}_1^{l} - (1+c) \times D_0 + D_1 = \begin{cases} 110 - (1+10\%) \cdot 140 + 62 & \text{if up} \\ 90 - (1+10\%) \cdot 140 + 62 & \text{if down} \end{cases}$$

and get



owner's cash flows

at state $\omega = d$ there is insolvency triggered by illiquidity.

Finally, let us evaluate the coupon rate for the "good" (non-insolvent) state $\omega = u$.

The creditors use the fundamental theorem. The fair value of their "investment" is $D_0 = 140$. What do they get in the case of complete transfer of assets?[1]

$$D_0 = \frac{\mathrm{E}_0^Q[\text{payments to creditors in } t=1]}{1+r_f} \qquad \text{by fundam. theorem}$$

$$140 = \frac{\overbrace{(1+c)\times 140}^{\text{no insolvency in } u}\times 0.0833 + \overbrace{(158.12+90)}^{\text{compl. transfer in } d}\times 0.9167}{1+10\%} \qquad \text{see}^2$$

$$c \approx -729.9\%$$

[1] Compare slide 8.

[2] The Q-probabilities are from lecture "Conclusions martingale-like CF". $158.12 + 90 = V_1(d) + CF_1)d$ and V_1 is from lecture "Cost of capital..."

What does this example show us? Remember: We assumed complete transfer of asset.

1. Either ensure that both insolvency triggers are met.
2. Or ensure that the coupon rates are not unrealistic.

We distinguish between illiquidity and over-indebtedness.

Over-indebtedness implies (later) illiquidity.

If complete transfer of assets is assumed either make sure that both triggers are met or that coupon rates are realistic.