

DCF: Firm income tax

Levered firms

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Let us have a closer look at levered firms: their value \tilde{V}_t^1 equals debt (\tilde{D}_t) plus equity (\tilde{E}_t)

$$\tilde{V}_t^1 = \tilde{D}_t + \tilde{E}_t.$$

These are **market values**, not book values! Debt is granted at time t , after one period the debtor has to pay redemption and interest \tilde{I}_{t+1} . **No default**

$$\tilde{I}_{t+1} = r_f \tilde{D}_t.$$

(Default will be discussed later.)



Debt ratio and leverage ratio are two important numbers. Both can be uncertain:

$$\left. \begin{array}{l} \text{debt ratio} \quad \tilde{l}_t = \frac{\tilde{D}_t}{\tilde{V}_t^1} \\ \text{leverage ratio} \quad \tilde{L}_t = \frac{\tilde{D}_t}{\tilde{E}_t} \end{array} \right\} \text{implies } \tilde{L}_t = \frac{\tilde{l}_t}{1 - \tilde{l}_t}.$$



Earnings before taxes are the tax base. Hence we have for the tax payments of the corporation

$$\widetilde{Tax} = \tau \widetilde{EBT}. \quad (*)$$

Because $\widetilde{EBIT}^u = \widetilde{EBIT}^l$, the tax payments of the levered and the unlevered firm differ by

$$\widetilde{Tax}_t^u - \widetilde{Tax}_t^l = \tau r_f \widetilde{D}_{t-1}.$$

This difference is called the **tax shield** from debt.



Interest is determined by the risk-free rate r_f and debt \tilde{D}_{t-1} . The tax shield is

$$\tau \cdot r_f \cdot \tilde{D}_{t-1}.$$

From our assumptions **only debt can be uncertain in this model!**

⇒ The level of tax shield is determined by the **financing policy**.

The aim of DCF is the **valuation of these (uncertain) tax advantages**. Not less, but also not more.



- 1 **Autonomous financing:** future amount of debt \tilde{D}_t is fixed.
- 2 **Financing based on market values:** evaluator sets the future debt ratios based on market values \tilde{l}_t .
- 3 **Financing based on book values:** the future debt ratios to book values \tilde{l}_t are fixed.
- 4 **Financing based on cash flows:** amount of debt is based on the firm's cash flows.
- 5 **Financing based on dividends:** debt managed so that previously determined dividend distributed.
- 6 **Financing based on dynamical leverage ratio:** evaluator sets the future cash flow–debt ratios.



We do not address which of these financing policies is most realistic. Nor do we discuss which of them maximizes the value of the levered firm (as we will see later, extending leverage increases firm value).

Assumption “given debt policy”: *The debt policy of the firm (although probably uncertain) is already prescribed.*



Our aim is a **general valuation equation** for the tax shield.

Free cash flows differ only by tax payments (see slide 4). Using the definition of tax base (*)

$$\widetilde{CF}_t^l = \widetilde{CF}_t^u + \tau r_f \widetilde{D}_{t-1}$$

From the fundamental theorem for levered and unlevered firms we get

$$\widetilde{V}_t^{l,u} = \frac{E_t^Q \left[\widetilde{CF}_{t+1}^{l,u} + \widetilde{V}_{t+1}^{l,u} \right]}{1 + r_f}.$$

It follows that

$$\widetilde{V}_t^l - \widetilde{V}_t^u = \frac{E_t^Q \left[\tau r_f \widetilde{D}_t + \widetilde{V}_{t+1}^l - \widetilde{V}_{t+1}^u \right]}{1 + r_f}.$$



This implies

$$\tilde{V}_t^l - \tilde{V}_t^u = \frac{E_t^Q [\tau r_f \tilde{D}_t]}{1 + r_f} + \frac{E_t^Q [\tilde{V}_{t+1}^l - \tilde{V}_{t+1}^u]}{(1 + r_f)^{T-t}}.$$

Now plugging the similar equation $\tilde{V}_{t+1}^l - \tilde{V}_{t+1}^u = \dots$ into this gives (“we see a pattern here”)

$$\tilde{V}_t^l - \tilde{V}_t^u = \frac{E_t^Q [\tau r_f \tilde{D}_t]}{1 + r_f} + \frac{E_t^Q [\tau r_f \tilde{D}_{t+1}]}{1 + r_f} + \frac{E_t^Q [\tilde{V}_{t+2}^l - \tilde{V}_{t+2}^u]}{(1 + r_f)^{T-t}}.$$

and we finally end up with



$$\tilde{V}_t^l = \tilde{V}_t^u + \frac{E_t^Q [\tau r_f \tilde{D}_t]}{1 + r_f} + \dots + \frac{E_t^Q [\tau r_f \tilde{D}_{T-1}]}{(1 + r_f)^{T-t}}.$$

or

$$\tilde{V}_t^l = \tilde{V}_t^u + \frac{\tau r_f E_t^Q [\tilde{D}_t]}{1 + r_f} + \dots + \frac{\tau r_f E_t^Q [\tilde{D}_{T-1}]}{(1 + r_f)^{T-t}}.$$

This is the **basic equation** for valuing the tax shield. It makes explicit that the tax shield depends on the financing policy, i.e., on future debt levels.



We consider levered and unlevered firms with identical gross cash flows.

The tax advantage for the levered firm (no default) is $\tau \cdot r_f \cdot \tilde{D}_{t-1}$.

The financing policy of the firm determines the value of the tax shield.

