

DCF: Firm income tax

Excursus: Excess volatility

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Is that typical for everyday life?

“We have ignored friction up to now...”

Almost any textbook on Newtonian Mechanics.



(source: the author)

So ignoring assumptions can be either useful or foolish, depending on the situation.



We formulated a heroic assumption (deterministic cost of capital), that is unrealistic:

$$\underbrace{E_t \left[\frac{\widetilde{CF}_{t+1}^u + \widetilde{V}_{t+1}^u}{\widetilde{V}_t^u} - 1 \right]}_{\text{expected return in } t} \text{ is deterministic}$$

How serious is this?



“The relevant question to ask about the ‘assumptions’ of a theory is not whether they are descriptively ‘realistic,’ for they never are, but whether they are sufficiently good approximations for the purpose in hand. And this question can be answered only by seeing whether the theory works, which means whether it yields sufficiently accurate predictions,”

Friedman (The methodology of positive economics 1953, p. 15).



Theorem (Williams, Gordon-Shapiro): *Assume costs of capital are deterministic and cash flows are martingale-like. Then*

$$\tilde{V}_t^u = \underbrace{\text{a number}}_{= \frac{1+g}{kE,^u - g}} \times \tilde{CF}_t^u$$

holds.

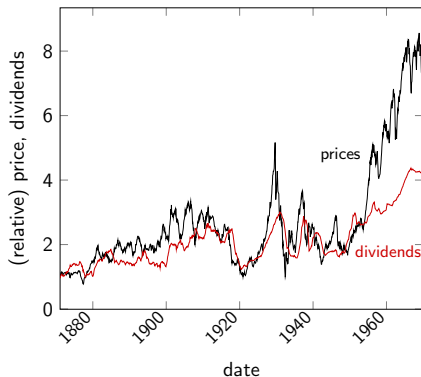
An observation: the “stochastic structure” of \tilde{V}_t^u and \tilde{CF}_t^u are **identical**.



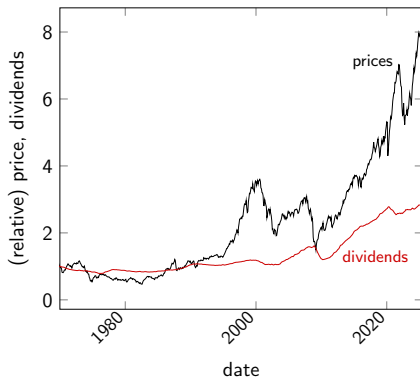
Stochastic structure

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These are statements about unlevered firms. We focus on stock market indices (US stock prices and dividends from www.shillerdata.com).



(a) From 1870-1969 (normalized to 1 in 1871).



(b) From 1970-2025 (normalized to 1 in 1970).



What do we see?

- 1 Values are far more volatile.
- 2 Dividends are “sticky”.

Excess volatility. Let us use the coefficient of variation

$$\frac{\text{Std}[\tilde{X}]}{E[\tilde{X}]}$$

to measure this:

$$\begin{array}{l} \text{dividends} \quad \frac{\text{Std}[\tilde{CF}^u]}{E[\tilde{CF}^u]} \approx 0.69 \\ \text{values} \quad \frac{\text{Std}[\tilde{V}^u]}{E[\tilde{V}^u]} \approx 1.28 \end{array}$$



However, this should not be the case:

$$\begin{aligned} \frac{\text{Std}[\widetilde{V}_t^u]}{\text{E}[\widetilde{V}_t^u]} &= \frac{\text{Std}[\text{number} \cdot \widetilde{CF}_t^u]}{\text{E}[\text{number} \cdot \widetilde{CF}_t^u]} && \text{by Gordon-Shapiro} \\ &= \frac{\text{number} \cdot \text{Std}[\widetilde{CF}_t^u]}{\text{number} \times \text{E}[\widetilde{CF}_t^u]} \\ &= \frac{\text{Std}[\widetilde{CF}_t^u]}{\text{E}[\widetilde{CF}_t^u]} \end{aligned}$$



We have several assumptions in our theory:

- Martingale-like cash flows
- deterministic cost of capital (“heroic”)
- even constant cost of capital
- ...

How do we know which one is important? **We can only guess.**



However, the heroic assumption is a major contender.

constant cost of capital?

“I have shown, however, that the movements in expected real interest rates that would justify the variability in stock prices are very large—much larger than the movements in nominal interest rates over the sample period,”
Shiller (The American Economic Review 1981, p. 434).

martingale-like? We also provided a comprehensive motivation for this assumption.



Suppose, for the sake of argument, that one could prove an equation of the form

$$\tilde{V}_t^u = \frac{\widetilde{CF}_t^u}{\kappa_t}$$

with a random and uncorrelated κ_t . Then the excess-volatility problem would be resolved.

While there is work along these lines¹, it would be overstating the case to call this idea established. We therefore end the discussion here.

¹<https://ssrn.com/abstract=3761127>