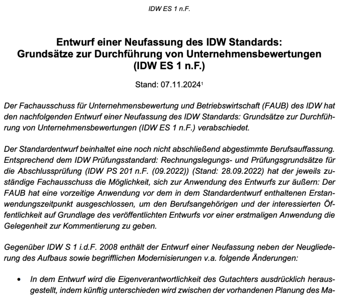
DCF: Firm income tax
Martingale like cash flows

Lutz Kruschwitz & Andreas Löffler

Freie Universität Berlin, Germany



IDW ES 1 n.F.

Entwurf einer Neufassung des IDW Standards:
Grundsätze zur Durchführung von Unternehmensbewertungen
(IDW ES 1 n.F.)

Stand: 07.11.2024[1]

Der Fachausschuss für Unternehmensbewertung und Betriebswirtschaft (FAUB) des IDW hat den nachfolgenden Entwurf einer Neufassung des IDW Standards: Grundsätze zur Durchführung von Unternehmensbewertungen (IDW ES 1 n.F.) verabschiedet.

Der Standardentwurf beinhaltet eine noch nicht abschließend abgestimmte Berufsauffassung. Entsprechend dem IDW Prüfungsstandard: Rechnungslegungs- und Prüfungsgrundsätze für die Abschlussprüfung (IDW PS 201 n.F. (09.2022)) (Stand: 28.09.2022) hat der jeweils zuständige Fachausschuss die Möglichkeit, sich zur Anwendung des Entwurfs zur äußern: Der FAUB hat eine vorzeitige Anwendung vor dem in dem Standardentwurf enthaltenen Erstanwendungszeitpunkt ausgeschlossen, um den Berufsangehörigen und der interessierten Öffentlichkeit auf Grundlage des veröffentlichten Entwurfs vor einer erstmaligen Anwendung die Gelegenheit zur Kommentierung zu geben.

Gegenüber IDW S 1 i.d.F. 2008 enthält der Entwurf einer Neufassung neben der Neugliederung des Aufbaus sowie begrifflichen Modernisierungen v.a. folgende Änderungen:

- *In dem Entwurf wird die Eigenverantwortlichkeit des Gutachters ausdrücklich herausgestellt, indem künftig unterschieden wird zwischen der vorhandenen Planung des Ma-*

Valuation standard of the German Association of CPAs (“IDW”).

We are not interested in past cash flows but in future ones.

These are uncertain.

We must estimate them.

So what is the right “narrative” to tell about the quantities?

And what follows for the stochastic structure of those cash flows?

Notice: We will only consider unlevered firms and their cash flows here.

If unlevered firms have a particular stochastic structure, the structure of the levered cash flows follows. (Later we will discuss levered firms.)

Let us try some ideas.[1]

Assume that future cash flows are independent of each other.

Before we set off, why is this economically likely to be nonsense?

What does "independent" mean here: The cash flow in year t has nothing to do with the cash flow in year $t+1$.

But then any corporate strategy would seem to have no effect. Pure chance would determine where the firm ends up.

[1] Spoiler: we begin with something deliberately misguided, to make our later assumption more plausible.

We need another rule that we did not introduce earlier.

"Rule 6" *If cash flows are independent, then* $\mathrm{E}_t^Q[\widetilde{CF}^u_{t+1}]$ *is not random but a number.*

Then, by using iterated expectation (rule 4), we get $(s > t)$

$$\mathrm{E}_t^Q[\widetilde{CF}^{\mathrm{u}}_s] = \mathrm{E}_t^Q[\underbrace{\mathrm{E}_{s-1}^Q[\widetilde{CF}^{\mathrm{u}}_s]}_{\text{is a number}}]$$

and by using rule 3 the expectation $\mathrm{E}_t^Q[\widetilde{CF}^{\mathrm{u}}_s]$ is a number as well.

This has implications for the value of the firm:

$$V_t^u = \frac{\overbrace{\mathrm{E}_t^Q[\widetilde{CF}^{\mathrm{u}}_{t+1}]}^{\text{number}}}{1+r_f} + \frac{\overbrace{\mathrm{E}_t^Q[\widetilde{CF}^{\mathrm{u}}_{t+2}]}^{\text{number}}}{(1+r_f)^2} + \ldots + \frac{\overbrace{\mathrm{E}_t^Q[\widetilde{CF}^{\mathrm{u}}_{T}]}^{\text{number}}}{(1+r_f)^{T-t-1}}$$

It is a number (deterministic!).

Independent cash flows means: "If future cash flows are driven purely by chance rather than strategy, then the firm's value is no longer random."

We will not make such an assumption.

Instead: Persistence and Shocks

We need an assumption

- under which the firm's strategy plays a role.
- Future cash flows should evolve from current cash flows plus shocks,
- while also allowing for possible growth.

We assume martingale-like cash flows.

Probably first introduced by Fama (1977):

> to zero. In short, rational assessment of expectations requires that the expected value of the earnings $\tilde{X}_t$ to be realized at the fixed time t evolves as a martingale.

However, Fama **does not** look at multiple cash flows in this section...

We assume

$$\mathrm{E}_t[\widetilde{CF}^{\mathrm{u}}_{t+1}] = (1+g) \cdot \widetilde{CF}^{\mathrm{u}}_t .$$

g is the growth rate of the cash flows ($g > -1$), cash flows are always positive.

This needs to be explained in more detail.

Consider a binomial tree of cash flows,

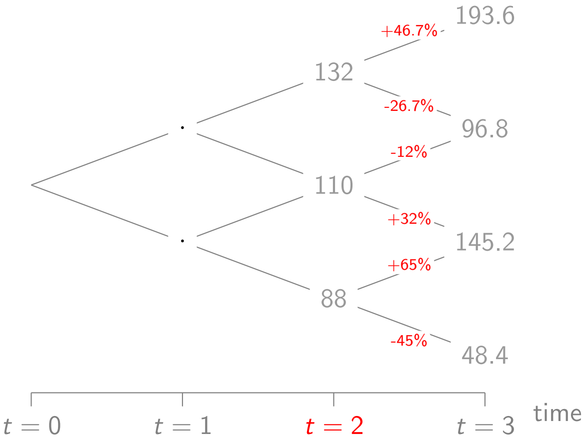


Figure: At each node, the expected growth rate is g = 10%.

Intuition of martingale-like

Loosely speaking, “martingale-like” cash flows mean that

- cash flows are generated by an underlying narrative (a “story”),
- cash flows are uncertain, but
- despite this uncertainty, their average (i.e., their expectation at each node) is anchored to the past value.

We have choosen the expectation under the subjective measure (we used $\mathrm{E}_t[\cdot]$) instead of the risk-neutral measure (we did not use $\mathrm{E}_t^Q[\cdot]$). Why?

It does not matter. In an exercise we show that the cash flows are martingale-like under Q as well, if cost of capital are deterministic.

Martingale-like and AR(1)

Martingale-like is linked to autoregressive variables (AR).

But autoregressive variables must have a *constant term* that is missing here, i.e., if cash flows would be AR(1) then

$$\mathrm{E}_t[\widetilde{CF}^{\mathrm{u}}_{t+1}] = \underbrace{\text{some constant} +}_{\text{missing here!}} (1+g)\widetilde{CF}^{\mathrm{u}}_t .$$

Define noise by

$$\widetilde{\varepsilon}_{t+1} := \widetilde{CF}^{\mathrm{u}}_{t+1} - (1+g)\widetilde{CF}^{\mathrm{u}}_{t}.$$

Then we can show

1. **Noise has no expectation**[2]

$$\mathrm{E}\left[\widetilde{\varepsilon}_t\right] = 0.$$

2. **Noise terms are uncorrelated**

$$\mathrm{Cov}\left[\widetilde{\varepsilon}_s, \widetilde{\varepsilon}_t\right] = 0 \qquad \text{if } s \neq t$$

[2] In the problems we show: That also holds for the conditional expectation.

Noise terms have no expectation:

$$\begin{aligned}
\mathsf{E}\left[\widetilde{\varepsilon}_{t+1}\right] &= \mathsf{E}_0\left[\widetilde{\varepsilon}_{t+1}\right] && \text{by rule 1}\\
&= \mathsf{E}_0\left[\mathsf{E}_t\left[\widetilde{\varepsilon}_{t+1}\right]\right] && \text{by rule 4}\\
&= \mathsf{E}_0\left[\mathsf{E}_t\left[\widetilde{CF}^{\mathrm{u}}_{t+1} - (1+g)\widetilde{CF}^{\mathrm{u}}_{t}\right]\right] && \text{by definition}\\
&= \mathsf{E}_0\left[\mathsf{E}_t\left[\widetilde{CF}^{\mathrm{u}}_{t+1}\right] - \mathsf{E}_t\left[(1+g)\widetilde{CF}^{\mathrm{u}}_{t}\right]\right] && \text{by rule 2}\\
&= \mathsf{E}_0\left[\mathsf{E}_t\left[\widetilde{CF}^{\mathrm{u}}_{t+1}\right] - (1+g)\widetilde{CF}^{\mathrm{u}}_{t}\right] && \text{by rule 5}\\
&= \mathsf{E}_0\left[(1+g)\widetilde{CF}^{\mathrm{u}}_{t} - (1+g)\widetilde{CF}^{\mathrm{u}}_{t}\right] && \text{by assumption}\\
&= \mathsf{E}_0[0] = 0 && \text{rule 3, QED}
\end{aligned}$$

Noise terms are uncorrelated: $(s < t)$

$$\begin{aligned}
\operatorname{Cov}[\widetilde{\varepsilon}_s, \widetilde{\varepsilon}_t] &= \mathrm{E}[\widetilde{\varepsilon}_s \widetilde{\varepsilon}_t] - \underbrace{\mathrm{E}[\widetilde{\varepsilon}_s]\,\mathrm{E}[\widetilde{\varepsilon}_t]}_{=0} && \text{by definition of covariance} \\
&= \mathrm{E}[\widetilde{\varepsilon}_s \widetilde{\varepsilon}_t] \\
&= \mathrm{E}_0[\widetilde{\varepsilon}_s \widetilde{\varepsilon}_t] && \text{by rule 1} \\
&= \mathrm{E}_0[\mathrm{E}_s[\widetilde{\varepsilon}_s \widetilde{\varepsilon}_t]] && \text{by rule 4} \\
&= \mathrm{E}_0\Big[\widetilde{\varepsilon}_s \underbrace{\mathrm{E}_s[\widetilde{\varepsilon}_t]}_{\substack{\text{will now shown}\\ \text{to be 0}}}\Big] && \text{by rule 5} \\
&= 0 && \text{rule 3, QED}
\end{aligned}$$

Proof (2) continued

$$
\begin{aligned}
\mathrm{E}_s\left[\tilde{\varepsilon}_t\right] &= \mathrm{E}_s\left[\widetilde{CF}^{\mathrm{u}}_{t+1} - (1+g_t)\widetilde{CF}^{\mathrm{u}}_t\right] \\
&= \mathrm{E}_s\left[\mathrm{E}_t\left[\widetilde{CF}^{\mathrm{u}}_{t+1} - (1+g_t)\widetilde{CF}^{\mathrm{u}}_t\right]\right] && \text{rule 4} \\
&= \mathrm{E}_s\left[\mathrm{E}_t\left[\widetilde{CF}^{\mathrm{u}}_{t+1}\right] - (1+g_t)\widetilde{CF}^{\mathrm{u}}_t\right] && \text{rule 2 and 5} \\
&= 0 && \text{by assumption}
\end{aligned}
$$

Martingale-like can be written as

$$\widetilde{CF}^{u}_{t+1} = (1+g)\widetilde{CF}^{u}_{t} + \widetilde{\varepsilon}_{t+1}.$$

In finance the usual assumption is: noise terms $\widetilde{\varepsilon}_t$ are pairwise **independent**. But here we only assume that the noise terms are pairwise uncorrelated – which is less restrictive:

$\widetilde{\varepsilon}_t, \widetilde{\varepsilon}_s$ are

uncorrelated if	independent if for all functions f and g
$\text{Cov}\left[\widetilde{\varepsilon}_t, \widetilde{\varepsilon}_s\right] = 0$	$\text{Cov}\left[f\left(\widetilde{\varepsilon}_t\right), g\left(\widetilde{\varepsilon}_s\right)\right] = 0$

An important remark 18

It seems **purely by chance** that

$$\mathrm{E}_1\left[\widetilde{CF}_3\right] = 1.1^{3-1} \times \widetilde{CF}_1\,,$$

but it is on purpose! This will become clear later (when discussing martingale-like cash flows).

We already hinted at it: in both cases of the example, the cash flows are martingale-like.

Let us verify this briefly.

The infinite example

$$\mathrm{E}_t[\widetilde{CF}^{\mathrm{u}}_{t+1}] = \frac{1}{2} u \widetilde{CF}^{\mathrm{u}}_t + \frac{1}{2} d \widetilde{CF}^{\mathrm{u}}_t = \underbrace{\frac{u+d}{2}}_{=1} \widetilde{CF}_t$$

The finite example: We always have

$$\mathrm{E}_t[\widetilde{CF}^{\mathrm{u}}_{t+1}] = 1.1 \cdot \widetilde{CF}^{\mathrm{u}}_t .$$

We will consider unlevered and levered firms.

Cash flows of the unlevered firm are martingale-like, i.e., noise terms are uncorrelated.