DCF: Firm income tax
Unlevered firms

Lutz Kruschwitz & Andreas Löffler

Freie Universität Berlin, Germany

The unlevered firm 2

Companies are indebted, i.e. levered. Why should we consider unlevered firms, i.e. firms without debt?

Valuation requires knowledge of

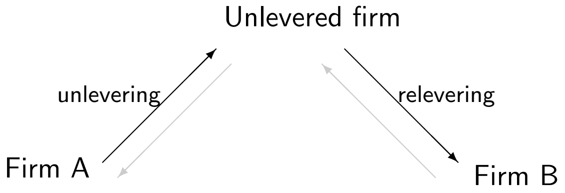
- cash flows $\Longleftarrow$ from business plans, annual balance sheets etc.
- taxes $\Longleftarrow$ from tax law
- cost of capital $\Longleftarrow$ from similar companies.

What is a "similar company"?

Companies are similar with respect to

- business risk
- financial risk (= different leverage ratio).

We eliminate the financial risk by determining the cost of capital of an unlevered firm (unlevering) and then of a levered firm (relevering).



levered firm	index l
unlevered firm	index u
free cash flows (after taxes)	$\widetilde{CF}_t^{\mathrm{u}}$
value	$\widetilde{V}_t^{\mathrm{u}}$
cost of capital	$k_t^{E,\mathrm{u}} =_{\mathsf{Def}} \frac{\mathsf{E}_t\left[\widetilde{CF}_{t+1}^{\mathrm{u}}+\widetilde{V}_{t+1}^{\mathrm{u}}\right]}{\widetilde{V}_t^{\mathrm{u}}} - 1$

Value of unlevered firm

We have, analogously to chapter 1 (even with the same proof!)

Theorem (value of unlevered firm): *With deterministic cost of capital of the unlevered firm*

$$\widetilde{V}_t^{\mathrm{u}} = \sum_{s=t+1}^{T} \frac{\mathsf{E}_t\left[\widetilde{CF}_s^{\mathrm{u}}\right]}{\left(1+k_t^{E,\mathrm{u}}\right)\cdots\left(1+k_{s-1}^{E,\mathrm{u}}\right)}.$$

We have established essentially the same result twice already; we now prove it a third time.

Reformulate our definition

$$\widetilde{V}_t^{\mathrm{u}} = \frac{\mathrm{E}_t\left[\widetilde{CF}_{t+1}^{\mathrm{u}} + \widetilde{V}_{t+1}^{\mathrm{u}}\right]}{1 + k_t^{E,\mathrm{u}}}.$$

Using the same relation next period

$$\widetilde{V}_{t+1}^{\mathrm{u}} = \frac{\mathrm{E}_{t+1}\left[\widetilde{CF}_{t+2}^{\mathrm{u}} + \widetilde{V}_{t+2}^{\mathrm{u}}\right]}{1 + k_{t+1}^{E,\mathrm{u}}},$$

the result is

$$\widetilde{V}_t^{\mathrm{u}} = \frac{\mathrm{E}_t\left[\widetilde{CF}_{t+1}^{\mathrm{u}} + \frac{\mathrm{E}_{t+1}\left[\widetilde{CF}_{t+2}^{\mathrm{u}} + \widetilde{V}_{t+2}^{\mathrm{u}}\right]}{1+k_{t+1}^{E,\mathrm{u}}}\right]}{1 + k_t^{E,\mathrm{u}}}.$$

Because the cost of capital of the unlevered company is deterministic, we can apply the Linearity Rule (no. 3) of conditional expectation and get

$$\widetilde{V}_t^{\mathrm{u}} = \frac{\mathrm{E}_t[\widetilde{CF}_{t+1}^{\mathrm{u}}] + \frac{\mathrm{E}_t\left[\mathrm{E}_{t+1}[\widetilde{CF}_{t+2}^{\mathrm{u}} + \widetilde{V}_{t+2}^{\mathrm{u}}]\right]}{1+k_{t+1}^{E,\mathrm{u}}}}{1 + k_t^{E,\mathrm{u}}}.$$

Using the Rule of Iterated Expectation (no. 4) we get

$$\widetilde{V}_t^{\mathrm{u}} = \frac{\mathrm{E}_t[\widetilde{CF}_{t+1}^{\mathrm{u}}] + \frac{\mathrm{E}_t\left[\widetilde{CF}_{t+2}^{\mathrm{u}} + \widetilde{V}_{t+2}^{\mathrm{u}}\right]}{1+k_{t+1}^{E,\mathrm{u}}}}{1 + k_t^{E,\mathrm{u}}}$$

Rearranging and using linearity again gives

$$\widetilde{V}_t^{\mathrm{u}} = \frac{\mathsf{E}_t[\widetilde{CF}^{\mathrm{u}}_{t+1}]}{1+k_t^{E,\mathrm{u}}} + \frac{\mathsf{E}_t\left[\widetilde{CF}^{\mathrm{u}}_{t+2}\right]}{(1+k_t^{E,\mathrm{u}})(1+k_{t+1}^{E,\mathrm{u}})} + \frac{\mathsf{E}_t\left[\widetilde{V}^{\mathrm{u}}_{t+2}\right]}{(1+k_t^{E,\mathrm{u}})(1+k_{t+1}^{E,\mathrm{u}})}$$

This procedure can be applied again and again: We see a pattern here!

If T is the last point in time where $\widetilde{V}_T = 0$, we get finally our first valuation equation.

$$\widetilde{V}_t^{\mathrm{u}} = \frac{\mathrm{E}_t[\widetilde{CF}^{\mathrm{u}}_{t+1}]}{1+k_t^{E,\mathrm{u}}} + \frac{\mathrm{E}_t\left[\widetilde{CF}^{\mathrm{u}}_{t+2}\right]}{(1+k_t^{E,\mathrm{u}})(1+k_{t+1}^{E,\mathrm{u}})} + \ldots + \frac{\mathrm{E}_t\left[\widetilde{CF}^{\mathrm{u}}_{T}\right]}{(1+k_t^{E,\mathrm{u}})\cdots(1+k_{T-1}^{E,\mathrm{u}})}$$

This is what we wanted to show.

We consider unlevered and levered firms.

Unlevered firms provide a natural benchmark for valuation.

A valuation equation for unlevered firms can be established.