DCF: Basic Concepts
Valuation

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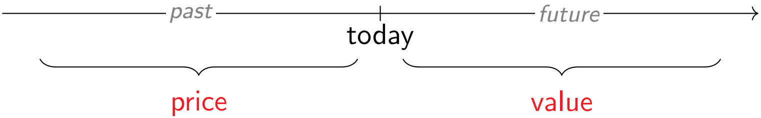
Value and prize

Some scholars draw a very precise distinction between "price" and "value":



They refer to different things

prize	value
unique/observable	can be ambiguous
number	random variable
no model	model/theory needed

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CAPITAL ASSET PRICES: A THEORY OF MARKET EQUILIBRIUM UNDER CONDITIONS OF RISK*

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I. Introduction

One of the problems which has plagued those attempting to predict the behavior of capital markets is the absence of a body of positive micro-economic theory dealing with conditions of risk. Although many useful

The CAP(ricing)M

Often, however, the two terms are used interchangeably. We will do the same from now on.

You should be aware, however, that this might be problematic.

Arbitrage and equilibrium

Prices[1] are explained using two different concepts:

Equilibrium argument One explains a price by referring to objects that typically do not have a price. For example: supply ("endowments") equals demand (determined by "utility functions").

Famous cases: Arrow-Debreu equilibrium, CAPM, Fisher model, Lucas model...

Arbitrage argument One explains a price by referring to another asset and to that asset's price. For example: Two assets with the same payments must have the same price.

Famous cases: Black-Scholes model, Put-Call parity, Modigliani-Miller theorems, Ross' APT...

DCF focuses solely on arbitrage.

[1] Shouldn't we say values?!...

How do we value in a **risk-free** world?

In a deterministic world that is **free of arbitrage** we must have

$$V_0 = \frac{CF_1}{1 + r_f}\ .$$

Otherwise, if for example

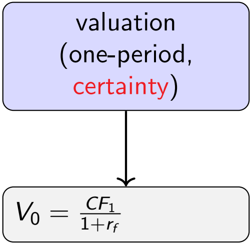
$$CF_1 > (1 + r_f)V_0$$

$\Longrightarrow$ take loan, buy share in $t = 0$, wait until $t = 1$, get dividend and sell share and pay back the loan plus interest

$\Longrightarrow$ get infinitely rich without any cost.

where V_0 value today, CF_1 cash flow tomorrow, r_f risk-free rate

Assumption (no free lunch) The capital market is free of arbitrage.



In a risky world we have

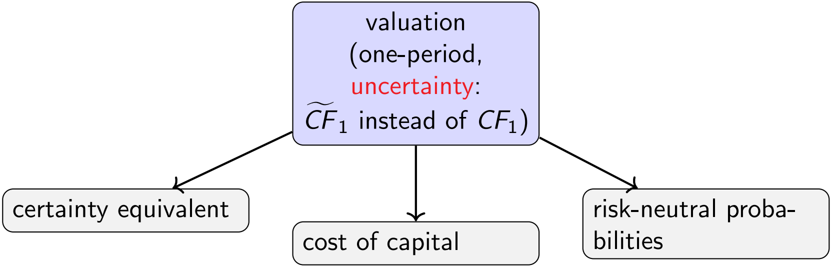
$$\widetilde{CF}_1 = \begin{cases} 110 & \text{up,} \\ 90 & \text{down.} \end{cases}$$

The value is now less then the risk-free value:

$$V_0 < \frac{\mathrm{E}\left[\widetilde{CF}_1\right]}{1 + r_f}.$$

Risky world

Assumption (no free lunch) The capital market is free of arbitrage.



We will now present these three approaches. They lead to the same value.

Requires a "utility function", for example $u(x) = \sqrt{x}$.

Consider cash flows of the form

$$\widetilde{CF}_1 = \begin{cases} 110 & \text{if up in } t = 1, \\ 90 & \text{if down in } t = 1. \end{cases}$$

Then certainty equivalent S_1 is

$$u(S_1) := \mathrm{E}[u(\widetilde{CF}_1)] = \frac{1}{2}\sqrt{110} + \frac{1}{2}\sqrt{90}$$

resulting in

$$u(S_1) \approx 9.988 \qquad \Longrightarrow \qquad S_1 \approx 99.75$$

The fair price is then given by ($r_f = 5\%$)

$$V_0 = \frac{S_1}{1 + r_f} = \frac{99.75}{1 + 5\%} = 95.00$$

Arbitrage: Cost of capital

Much more widespread in practice. Here, we take the expected value and add a risk premium z of 0.264% in the denominator:

$$V_0 = \frac{\mathrm{E}[\widetilde{CF}_1]}{1 + r_f + z} = \frac{0.5 \times 110 + 0.5 \times 90}{1 + 5\% + 0.264\%} = 95$$

Used mainly in option pricing but not in valuation so often. Here, we change the probabilities to $Q(u), Q(d)$. Up is now 48.75% and we get

$$V_0 = \frac{\mathrm{E}^{Q}[\widetilde{CF}_1]}{1+r_f} = \frac{0.4875 \times 110 + 0.5125 \times 90}{1+5\%} = 95$$

Three roads lead to Rome

Certainty equivalent

$$\frac{\mathsf{E}_t\left[\widetilde{CF}_{t+1}+\widetilde{V}_{t+1}\right]-\text{risk adjustment}}{1+r_f}$$

Risk premium

$$\frac{\mathsf{E}_t\left[\widetilde{CF}_{t+1}+\widetilde{V}_{t+1}\right]}{1+r_f+\text{risk premium}}$$

Risk-neutral probability

$$\frac{\mathsf{E}_t^Q\left[\widetilde{CF}_{t+1}+\widetilde{V}_{t+1}\right]}{1+r_f}$$

Summary

Price and value are different concepts.

DCF builds on arbitrage free markets, not equilibrium.

DCF focuses on risk-neutral probabilities.