

## Lecture: Cost of Capital and Tax Rate

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Discounted Cash Flow, Section 3.2



# Outline

## 3.2 Excursus: cost of capital and tax rate

- The problem

- Usually stated

- An arbitrage opportunity

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- Summary



# The problem

Up to now we have looked for the firm's value **given the tax rate**.

Now we ask for a **varying tax rate**  $\tau$ .

This boils down to the question of how cost of capital  $k^{E,u}$  changes with  $\tau$ .

(Remember:  $k^{E,u}$  **is post-tax!**)



# Usually stated (Johansson 1969)

affected by the presence of investor taxes. Let  $\tau_i$  be the tax rate investors pay on equity income (dividends) and  $\tau_d$  be the tax rate investors pay on interest income. Then, given an expected return on debt  $r_D$ , define  $r_D^*$  as the expected return on equity income that would give investors the same after-tax return:

$$r_D^* (1 - \tau_i) = r_D (1 - \tau_d)$$

So

$$r_D^* = r_D \frac{(1 - \tau_d)}{(1 - \tau_i)} \quad (18.23)$$

Because the unlevered cost of capital is for a hypothetical firm that is all equity,

Berk/DeMarzo: Corporate  
Finance, 2007

The literature on valuation suggests a relation between cost of equity post-tax  $k^{E,u}$  and tax rate  $\tau$  where

$$k^{E,u} = k^E (1 - \tau). \quad (3.6)$$

$k^E$  is sometimes interpreted as 'cost of capital before-tax'.

Important is only the linearity: For example, increasing the tax rate from 0% to 50% lowers cost of capital by one half.

Nevertheless, **this equation is very problematic.**



# Example

Look at a company that

- ▶ lives infinitely,
- ▶ has constant expected cash flows,
- ▶ no retainments and no investments.

For such a firm

$$\widetilde{FCF}_t^u = \widetilde{GCF}_t(1 - \tau) \quad (3.5)$$

holds, which is very convenient.

(Assumptions has to be made about gross instead of free cash flows because the tax rate will change.)



# Valuation equation

Then

$$\tilde{V}_t = \frac{\widetilde{FCF}_t^u}{k^{E,u}}$$

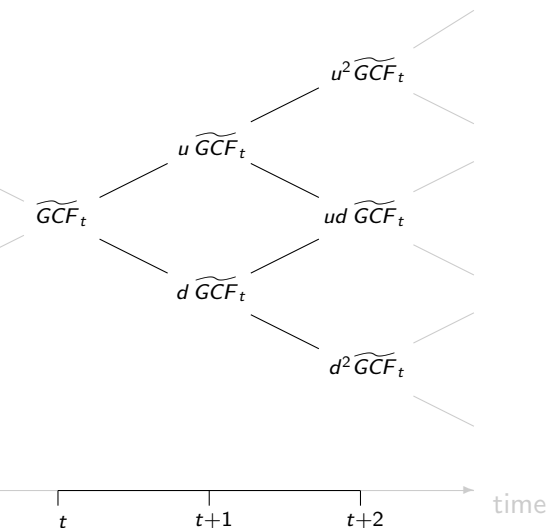
and from (3.5) with (3.6)

$$\tilde{V}_t = \frac{\widetilde{FCF}_t^u}{k^{E,u}} = \frac{\widetilde{GCF}_t(1-\tau)}{k^E(1-\tau)} = \frac{\widetilde{GCF}_t}{k^E}. \quad (3.7)$$

**The personal income tax rate cancels!** Personal taxes do not seem to have an influence on company value.



# Stochastic structure of gross cash flows



Consider our infinite example with gross cash flows following up with subjective probability  $P(u)$ , down with  $P(d)$ .



# Free cash flows weak autoregressive

Gross cash flows (before tax!) are weak autoregressive.

Are free cash flows (post tax!) weak autoregressive as well?

$$\begin{aligned} E \left[ \widetilde{FCF}_{t+1}^u | \mathcal{F}_t \right] &= E \left[ (1 - \tau) \widetilde{GCF}_{t+1} | \mathcal{F}_t \right] \\ &= (1 - \tau) E \left[ \widetilde{GCF}_{t+1} | \mathcal{F}_t \right] \\ &= (1 - \tau) P(u) u \widetilde{GCF}_t + (1 - \tau) P(d) d \widetilde{GCF}_t \\ &= \underbrace{\left( P(u) u + P(d) d \right)}_{:= 1+g} (1 - \tau) \widetilde{GCF}_t \\ &= (1 + g) \widetilde{FCF}_t^u. \end{aligned}$$

Yes!





# The market

Now consider **two firms**

	firm A	firm A'
up and down factor	$u, d$	$u', d'$
gross cash flows	$\widetilde{GCF}_t$	$\widetilde{GCF}'_t$
firm values	$\widetilde{V}_t$	$\widetilde{V}'_t$
cost of capital	$k$	$k'$
growth rate	$g \stackrel{!}{=} 0$	$g' \stackrel{!}{=} 0$

The up- and down-movements of both cash flows are perfectly correlated with probability  $P(u)$  and  $P(d)$ .



# Duplication

There is one riskless bond with value  $B_t$  at time  $t$ . The riskless interest rate after tax is  $r_f(1 - \tau)$ . We now duplicate the payments of firm  $A'$  by a **portfolio of A and bond**.

This portfolio contains

$n_B :=$  bonds and

$n_A :=$  shares of firm A

such that its payments equal the dividend of  $A'$ . Or,

$$\begin{aligned} n_B B_t (1 + r_f(1 - \tau)) + n_A \left( \widetilde{GCF}_{t+1}(1 - \tau) + \widetilde{V}_{t+1} \right) \\ = \widetilde{GCF}'_{t+1}(1 - \tau) + \widetilde{V}'_{t+1}. \end{aligned}$$



# Duplication

To determine  $n_A$  and  $n_B$  we use (3.7) and this gives

$$\begin{aligned}n_B B_t (1 + r_f(1 - \tau)) + n_A (1 + k_{t+1}(1 - \tau)) \tilde{V}_{t+1} \\ = (1 + k'_{t+1}(1 - \tau)) \tilde{V}'_{t+1}\end{aligned}$$

or, given the stochastic structure,

$$\begin{aligned}n_B (1 + r_f(1 - \tau)) B_t + n_A (1 + k(1 - \tau)) u \tilde{V}_t &= (1 + k'(1 - \tau)) u' \tilde{V}'_t \\ n_B (1 + r_f(1 - \tau)) B_t + n_A (1 + k(1 - \tau)) d \tilde{V}_t &= (1 + k'(1 - \tau)) d' \tilde{V}'_t.\end{aligned}$$



# Duplication

This is a  $2 \times 2$ -system that can easily be solved:

$$n_B = \frac{\tilde{V}'_t}{B_t} \frac{(u - u')(1 + k'(1 - \tau))}{u(1 + r_f(1 - \tau))}$$
$$n_A = \frac{\tilde{V}'_t}{\tilde{V}_t} \frac{u'(1 + k'(1 - \tau))}{u(1 + k(1 - \tau))}.$$

(All variables are uncertain.)

Furthermore, since the market is free of arbitrage, we must have

$$n_B B_t + n_A \tilde{V}_t = \tilde{V}'_t.$$



# Duplication

There are now three equations. Plugging them all together results in

$$\frac{u - u'}{1 + r_f(1 - \tau)} + \frac{u'}{1 + k(1 - \tau)} = \frac{u}{1 + k'(1 - \tau)} \quad (3.10)$$

and this is a relation between

- the cost of capital  $k$ ,  $k'$  and  $r_f$  before taxes,
- the tax rate  $\tau$ , and
- the parameters  $u$  and  $u'$ .



# Duplication

Equation (3.10) is a **no arbitrage**-condition. If it is not satisfied there is an arbitrage opportunity in the market.

But: It is also a quadratic equation and such an equation has only two solutions. These are

$$\tau = 100\% \text{ and}$$

$$\tau = 0\%.$$

For any other tax rate **there must be an arbitrage opportunity**. This violates our basic principle of valuation.



# Intuition of the result

Our result is in fact surprising. Is there any intuition for it?

Notice that cost of capital  $k^{E,u}$  and company value  $\tilde{V}_t$  are related to each other (like “two sides of a coin”). By determining a relation between cost of capital and tax rate we implicitly determine a relation between value and tax rate.

**But this relation is highly non-linear** which is the reason for our arbitrage opportunity.



# Summary

**Never ever use**

$$k^{E,u} = k^E(1 - \tau)$$

**when the tax rate  $\tau$  changes.**

