Lecture: Personal Income Tax

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Discounted Cash Flow, Section 3.1

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Outline

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The pros and the cons

Why should we incorporate personal income tax?

- Personal income tax influences consumption.
- German CPAs take personal tax into account since 1997.

CPAs in other countries are more hesitant. Typically:
"Incorporation of personal income tax is not necessary because it cancels."

Unlevered and levered

In Chapter 2 (Corporate Income Tax) we used the terms

- *unlevered* = not indebted,
- *levered* = indebted.

This was appropriate because

- low debt implies high tax burden,
- high debt **leverages** low tax burden.

Continue former designation?

Now we are looking at **personal income tax**. Does the tax burden depend on indebtedness?

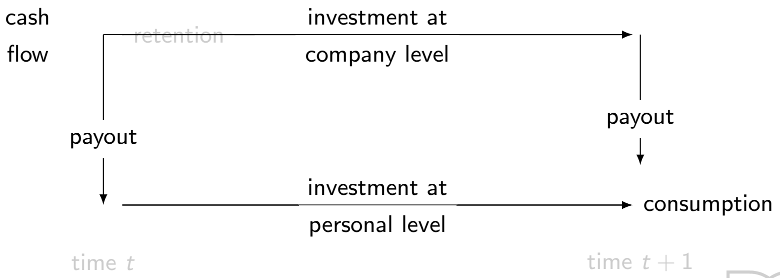
No, if riskless interest and dividends are taxed with the same rate (which we will assume for the moment): The personal tax of the shareholder – given the distribution – is independent of debt.

But now, the distribution policy plays a big role.

New interpretation

Our focus lies on the tax burden of shareholders. And here we notice that

- full distribution implies high tax burden,
- partial distribution **leverages** low tax burden.



Debt and leverage

The levered and the unlevered firm in this chapter are completely self-financed (no debt):

Personal Income Tax	(Chap. 3)
unlevered	**full distribution**
levered	**partial distribution**
further assump.	self-financed

New interpretation of variables

$\widetilde{V}_t^{\mathrm{u}}$	Market value with full distribution
$\widetilde{V}_t^{\mathrm{l}}$	Market value with partial distribution
$\widetilde{FCF}_t^{\mathrm{u}}$	Free cash flow with full distribution
$\widetilde{FCF}_t^{\mathrm{l}}$	Free cash flow with partial distribution
$\widetilde{Tax}_t^{\mathrm{u}}$	Personal income tax with full distribution
$\widetilde{Tax}_t^{\mathrm{l}}$	Personal income tax with partial distribution

Positive dividends

With corporate income tax negative cash flows meant nothing more than that the financiers infused the company with further equity. (If this was no longer possible we spoke of default.)

But negative payments do not make any sense in the case of dividends. Thus we will presuppose that the levered and the unlevered firm's cash flows are large enough so that the dividends **cannot turn negative**.

Unlevered and levered firms

To value a levered firm we need to value an unlevered firm.

This remains true in the case of firm income tax as well as in the case of personal income tax.

Definition of cost of capital of unlevered firm as in Chapter 2, i.e.

$$\widetilde{k}_t^{E,\mathrm{u}} := \frac{\mathsf{E}\left[\widetilde{FCF}^{\mathrm{u}}_{t+1} + \widetilde{V}^{\mathrm{u}}_{t+1} | \mathcal{F}_t\right]}{\widetilde{V}^{\mathrm{u}}_t} - 1 \ .$$

Valuation equation, weak autoregressive cash flows

Then, with deterministic cost of capital (without proof)

$$\widetilde{V}_t^{\mathrm{u}} = \sum_{s=t+1}^{T} \frac{\mathsf{E}\left[\widetilde{FCF}_s^{\mathrm{u}} | \mathcal{F}_t\right]}{\left(1 + k_t^{E,\mathrm{u}}\right) \dots \left(1 + k_{s-1}^{E,\mathrm{u}}\right)} \; .$$

Again we assume

$$\mathsf{E}\left[\widetilde{FCF}_{t+1}^{\mathrm{u}} | \mathcal{F}_t\right] = (1 + g_t) \widetilde{FCF}_t^{\mathrm{u}} \; .$$

Personal income tax

The characteristics of personal income tax are:

- The tax subject is a shareholder (and creditor).
- The tax base (categories of income) is for
  - shareholder: **dividends**, business income, ...
    (capital repayment exempt from taxes)
  - debt holder: **interest**.
- Notice, that we allow the tax rate to be different for dividends (τ^D) and interest (τ^I). This is the reason for considering creditors...

From gross cash flow to tax base

Pre-tax gross cash flow	$\widetilde{GCF}_t$
$-$ Investment expenses	$\widetilde{Inv}_t$
$=$ Shareholder's unlevered taxable income	$\widetilde{GCF}_t - \widetilde{Inv}_t$
$-$ Retained earnings	$\widetilde{A}_t$
$+$ Reflux from retained earnings	$(1+\widetilde{r}_{t-1})\widetilde{A}_{t-1}$
$=$ Shareholder's levered taxable income	$\widetilde{GCF}_t - \widetilde{Inv}_t$
	$-\widetilde{A}_t + (1+\widetilde{r}_{t-1})\widetilde{A}_{t-1}$

Interest on retainment

How is the retained amount invested?

Machines, tools, buildings,...?

No! All projects with positive NPV are already realized.

Debt repayment?

No! Debt schedule is already given.

Capital market?

Yes! But at what interest rate??

- riskless

$$\widetilde{r}_t = r_f$$

- risky

$$\widetilde{A}_t = \frac{\mathrm{E}_Q\left[(1+\widetilde{r}_t)\widetilde{A}_t|\mathcal{F}_t\right]}{1+r_f} \Longrightarrow \mathrm{E}_Q\left[\widetilde{r}_t|\mathcal{F}_t\right] = r_f$$

Tax equations

The tax equations are

for the unlevered company

$$\widetilde{Tax}_t^{\mathrm{u}} = \tau^D(\widetilde{GCF}_t - \widetilde{Inv}_t)$$

for the levered company

$$\widetilde{Tax}_t^{\mathrm{l}} = \tau^D \left(\widetilde{GCF}_t - \widetilde{Inv}_t - \widetilde{A}_t + (1 + \widetilde{r}_{t-1}) \widetilde{A}_{t-1} \right).$$

The tax payments of a levered company can be written as

$$\widetilde{Tax}_t^{\mathrm{l}} - \widetilde{Tax}_t^{\mathrm{u}} = \tau^D \Big(- \widetilde{A}_t + (1 + \widetilde{r}_{t-1}) \widetilde{A}_{t-1} \Big).$$

Tax difference of levered firm

We realize that the tax payments of the levered firm differ.
Although unlevered and levered firms have identical

- gross cash flows,
- stock depreciation and appreciation,
- debt schedule and
- investments

they differ in

- dividends and
- tax payments.

Tax advantage of levered firm

If dividends are not fully distributed, the levered firm pays **less** personal income taxes than the unlevered firm. There is a

(personal income) **tax shield for the levered firm –** justifing our designation. The tax differences are

$$\widetilde{FCF}^{\mathrm{l}}_t - \widetilde{FCF}^{\mathrm{u}}_t = \left(\ldots - \widetilde{A}_t + (1+\widetilde{r}_{t-1})\widetilde{A}_{t-1} - \widetilde{Tax}^{\mathrm{l}}_t\right) - \left(\ldots - \widetilde{Tax}^{\mathrm{u}}_t\right)$$
$$= (1-\tau^D)\left(-\widetilde{A}_t + (1+\widetilde{r}_{t-1})\widetilde{A}_{t-1}\right)$$

and after taking expectations

$$\mathrm{E}_Q\left[\widetilde{FCF}^{\mathrm{l}}_t - \widetilde{FCF}^{\mathrm{u}}_t | \mathcal{F}_{t-1}\right] = (1-\tau^D)\left((1+r_f)\widetilde{A}_{t-1} - \mathrm{E}_Q\left[\widetilde{A}_t | \mathcal{F}_{t-1}\right]\right).$$

Remember: Fundamental theorem

What is the fundamental theorem **without any tax**?

If capital markets are free of arbitrage, risk–neutral probabilities Q exist such that

$$V_t = \frac{\mathrm{E}_Q\left[\widetilde{FCF}_{t+1} + \widetilde{V}_{t+1}|\mathcal{F}_t\right]}{1 + r_f}.$$

What changed with **firm income tax**?

Only payments $\widetilde{FCF}_{t+1}$ were affected.

Fundamental theorem with personal income taxes

What changes will there be with **personal income tax**?

Both payments $\widetilde{X}_{t+1}$ and interest r_f will be affected.

But how?! We do not prove here that

$$\widetilde{V}_t = \frac{\mathrm{E}_Q\left[\widetilde{FCF}_{t+1} + \widetilde{V}_{t+1} | \mathcal{F}_t\right]}{1 + r_f(1 - \tau^I)}$$

holds.

Gordon-Shapiro again

Gordon-Shapiro with personal income tax,

$$\widetilde{V}_t^{\mathrm{u}} = \frac{\widetilde{FCF}_t^{\mathrm{u}}}{d_t^u}.$$

Cost of equity of unlevered firm is suitable as discount rates,

$$\frac{\mathrm{E}_Q\left[\widetilde{FCF}_s^{\mathrm{u}} | \mathcal{F}_t\right]}{(1 + r_f(1-\tau^I))^{s-t}} = \frac{\mathrm{E}\left[\widetilde{FCF}_s^{\mathrm{u}} | \mathcal{F}_t\right]}{\left(1 + k_t^{E,\mathrm{u}}\right) \dots \left(1 + k_{s-1}^{E,\mathrm{u}}\right)}.$$

Remark: Cost of equity $k^{E,\mathrm{u}}$ **after income tax**!

Differences in levered and unlevered firms

We know that the levered firm pays less taxes than the unlevered. But what is the value of this tax shield? And **what does this value depend upon?**

To this end we will assume that the last retainment at $T-1$ is zero, i.e. $\widetilde{A}_T = 0$.

Market values

The market value of the unlevered firm is

$$\widetilde{V}_t^{\mathrm{u}} = \frac{\mathsf{E}_Q\left[\widetilde{FCF}^{\mathrm{u}}_{t+1}|\mathcal{F}_t\right]}{1+r_f(1-\tau^I)} + \ldots + \frac{\mathsf{E}_Q\left[\widetilde{FCF}^{\mathrm{u}}_{T}|\mathcal{F}_t\right]}{(1+r_f(1-\tau^I))^{T-t}}.$$

The market value of the levered firm is

$$\widetilde{V}_t^{\mathrm{l}} = \frac{\mathsf{E}_Q\left[\widetilde{FCF}^{\mathrm{l}}_{t+1}|\mathcal{F}_t\right]}{1+r_f(1-\tau^I)} + \ldots + \frac{\mathsf{E}_Q\left[\widetilde{FCF}^{\mathrm{l}}_{T}|\mathcal{F}_t\right]}{(1+r_f(1-\tau^I))^{T-t}}.$$

Difference of market values

From both equations

$$\widetilde{V}_t^{\mathrm{l}} = \widetilde{V}_t^{\mathrm{u}} + \frac{(1-\tau^D)\,\mathrm{E}_Q\left[(1+r_f)\widetilde{A}_t - \widetilde{A}_{t+1}|\mathcal{F}_t\right]}{1+r_f(1-\tau^I)} + \dots$$

$$+ \frac{(1-\tau^D)\,\mathrm{E}_Q\left[(1+r_f)\widetilde{A}_{T-2} - \widetilde{A}_{T-1}|\mathcal{F}_t\right]}{1+r_f(1-\tau^I)^{T-t-1}}$$

$$+ \frac{(1-\tau^D)\,\mathrm{E}_Q\left[(1+r_f)\widetilde{A}_{T-1} \overbrace{-\widetilde{A}_T}^{=0} |\mathcal{F}_t\right]}{(1+r_f(1-\tau^I))^{T-t}}.$$

Difference of market values (continued)

Rearranging,

$$
\begin{aligned}
\widetilde{V}_t^{\mathrm{l}} = \widetilde{V}_t^{\mathrm{u}} &+ \frac{(1-\tau^D)(1+r_f)\widetilde{A}_t}{1+r_f(1-\tau^I)} \\
&+ \frac{\mathrm{E}_Q\left[\frac{(1+r_f)(1-\tau^D)}{1+r_f(1-\tau^I)}\widetilde{A}_{t+1} - (1-\tau^D)\widetilde{A}_{t+1}|\mathcal{F}_t\right]}{1+r_f(1-\tau^I)} + \dots \\
&+ \frac{\mathrm{E}_Q\left[\frac{(1+r_f)(1-\tau^D)}{1+r_f(1-\tau^I)}\widetilde{A}_{T-1} - (1-\tau^D)\widetilde{A}_{T-1}|\mathcal{F}_t\right]}{(1+r_f(1-\tau^I))^{T-1-t}}.
\end{aligned}
$$

Difference of market values (continued)

Which gives

$$\widetilde{V}_t^{\mathrm{l}} = \widetilde{V}_t^{\mathrm{u}} + \left(1-\tau^D\right)\widetilde{A}_t + \frac{\tau^I\left(1-\tau^D\right) r_f\, \mathsf{E}_Q\left[\widetilde{A}_t|\mathcal{F}_t\right]}{1+r_f\left(1-\tau^I\right)}$$

$$+\frac{\tau^I\left(1-\tau^D\right) r_f\, \mathsf{E}_Q\left[\widetilde{A}_{t+1}|\mathcal{F}_t\right]}{\left(1+r_f\left(1-\tau^I\right)\right)^2} + \ldots + \frac{\tau^I\left(1-\tau^D\right) r_f\, \mathsf{E}_Q\left[\widetilde{A}_{T-1}|\mathcal{F}_t\right]}{\left(1+r_f\left(1-\tau^I\right)\right)^{T-t}}.$$

Value of tax shield

Compare this to the firm income tax:

$$\widetilde{V}_t^{\mathrm{l}} = \widetilde{V}_t^{\mathrm{u}} + \frac{\tau r_f \operatorname{E}_Q\left[\widetilde{D}_t|\mathcal{F}_t\right]}{1+r_f} + \ldots + \frac{\tau r_f \operatorname{E}_Q\left[\widetilde{D}_{T-1}|\mathcal{F}_t\right]}{(1+r_f)^{T-t}}.$$

Although a different economic story, a similar formal structure! Only $\tau^I(1-\tau^D) r_f \widetilde{A}_t$ replaces $\tau r_f \widetilde{D}_t$.

General procedure

We proceed as in Chapter 2:

1. We formulate different distribution policies.
2. We modify the main valuation equation with this distribution.

Different retainment policies

With **firm income tax** the **debt schedule** was important, with **personal income tax** the **retainment schedule** is important. We will look at

1. retainment based on cash flow,
2. retainment based on dividends,
3. retainment based on market values.

Summary

Levered and unlevered for personal income tax means partial and full distribution.

The levered firm has a tax advantage.

The fundamental theorem holds with an post-tax riskless rate.

The distribution policy determines the value of the tax shield.