Lecture: Financing Based on Book Values

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Discounted Cash Flow, Section 2.5

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Outline

The problem 1

WACC is one of the most popular methods of evaluating a firm. But: it requires a deterministic leverage **to market values**, or

- if share price goes up $\longrightarrow$ debt has to go up as well and
- if share price goes down $\longrightarrow$ debt goes down as well and
- if share price goes up/down $\longrightarrow$ debt goes up/down as well.

This assumption seems **very unrealistic**. But what happens with the valuation formula if we replace the market values by **book values**?

That is the aim of this section!

The assumptions 2

Let us talk about book values. We assume for the book value of debt

$$\underline{\widetilde{D}}_t = \widetilde{D}_t$$

and no default or $\widetilde{I}_{t+1} = r_f \widetilde{D}_t$.

But that does not make sense with equity: usually $\underline{\widetilde{E}}_t \neq \widetilde{E}_t$! What drives the book value of equity?

What influences the book value of equity $\underline{\widetilde{E}}_t$? Three elements are possible:

1. Increase of subscribed capital (which we assume to be zero).
2. Retained earnings after tax given by $\left(\widetilde{EBIT}_{t+1} - \widetilde{I}_{t+1}\right)(1-\tau)$.
3. Paid dividends given by cash flows to shareholders, or $\widetilde{FCF}^{\mathrm{l}}_{t+1} - \left(\widetilde{I}_{t+1} + \widetilde{D}_t - \widetilde{D}_{t+1}\right)$.

Clean surplus relation 4

Hence, the book value of equity has to satisfy

$$\begin{aligned}\underline{\widetilde{E}}_{t+1} = & \underline{\widetilde{E}}_t + \left(\widetilde{EBIT}_{t+1} - \widetilde{I}_{t+1}\right)(1-\tau) \\ & - \left(\widetilde{FCF}^{\mathrm{u}}_{t+1} + \tau \widetilde{I}_{t+1} - \left(\widetilde{D}_t + \widetilde{I}_{t+1} - \widetilde{D}_{t+1}\right)\right)\end{aligned}$$

which can be simplified to

$$\underline{\widetilde{E}}_{t+1} = \underline{\widetilde{E}}_t + \widetilde{EBIT}_{t+1}(1-\tau) - \left(\widetilde{FCF}^{\mathrm{u}}_{t+1} - \left(\widetilde{D}_t - \widetilde{D}_{t+1}\right)\right).$$

Assumption 2.7 (clean surplus relation): *The book value satisfies*

$$\underline{\widetilde{E}}_{t+1} + \widetilde{D}_{t+1} = \underline{\widetilde{E}}_t + \widetilde{D}_t + \widetilde{EBIT}_{t+1}(1-\tau) - \widetilde{FCF}^{\mathrm{u}}_{t+1}.$$

From this follows our next theorem.

Theorem 2.13 (operating assets relation): *The book value of the firm follows*

$$\underbrace{\widetilde{\underline{V}}^{\mathrm{l}}_{t+1} = \widetilde{\underline{V}}^{\mathrm{l}}_{t}}_{\text{levered firm}} + \underbrace{\widetilde{EBIT}_{t+1}(1-\tau) - \widetilde{FCF}^{\mathrm{u}}_{t+1}}_{\text{from unlevered firm}}.$$

This relation invokes elements from the levered as well as from the unlevered firm.

Since the cash flows and EBIT are random, the book value will be a random variable as well.

Definition 2.9 (financing based on book values): *A company is financed based on book values if debt ratios to book values* $\widetilde{\underline{l}}_t$ *are deterministic.*

To get an idea of the company's value more information is necessary: we need to know about **investment and accruals** since they drive the book value.

Let us go back to Table 1.1:

$=$	Earnings before taxes	$\widetilde{EBT}$
$+$	Interest	$\widetilde{I}$
$=$	Earnings before interest and taxes	$\widetilde{EBIT}$
$+$	Accruals	$\widetilde{Accr}$
$=$	**Gross cash flow before taxes**	$\widetilde{GCF}$
$-$	Taxes	$\widetilde{Tax}$
$-$	Investment expenses	$\widetilde{Inv}$
$=$	Free cash flow	$\widetilde{FCF}^{\mathrm{l}}$

Hence (after some rearrangements),

$$\widetilde{FCF}^{\mathrm{u}}_{t+1} + \tau \widetilde{EBIT}_{t+1} + \widetilde{Inv}_{t+1} = \widetilde{GCF}_{t+1} = \widetilde{EBIT}_{t+1} + \widetilde{Accr}_{t+1}.$$

Hence,

$$\widetilde{EBIT}_{t+1}(1-\tau) - \widetilde{FCF}^{\mathrm{u}}_{t+1} = \widetilde{Inv}_{t+1} - \widetilde{Accr}_{t+1}. \qquad (2.21)$$

The rhs of (2.21) depends on the investment policy and accruals,

$$\widetilde{EBIT}_{t+1}(1-\tau) - \widetilde{FCF}^{\mathrm{u}}_{t+1} = \widetilde{Inv}_{t+1} - \widetilde{Accr}_{t+1}. \tag{2.21}$$

Now three investment policies will be differentiated:

1. investments due to full distribution;
2. replacement investments;
3. investment based on cash flows.

The profit after interest and taxes is

$$\left(\widetilde{EBIT}_{t+1} - \widetilde{I}_{t+1}\right)(1-\tau).$$

Owners receive

$$\widetilde{FCF}^{\mathrm{u}}_{t+1} + \tau \widetilde{I}_{t+1} - \left(\widetilde{D}_t + \widetilde{I}_{t+1} - \widetilde{D}_{t+1}\right).$$

In case of a full distribution policy both amounts are identical. Hence,

Definition 2.10 (full distribution policy): *A levered firm carries out a full distribution policy if*

$$\widetilde{FCF}^{\mathrm{u}}_{t+1} = \widetilde{EBIT}_{t+1}(1-\tau) + \underline{\widetilde{D}}_t - \underline{\widetilde{D}}_{t+1}.$$

It is doubtful whether full distribution is a realistic behaviour, but it has a strong tradition in Germany.

It is equivalent to investments completely financed by debt:

$$\widetilde{Inv}_{t+1} - \widetilde{Accr}_{t+1} = -\left(\underline{\widetilde{D}}_t - \underline{\widetilde{D}}_{t+1}\right)$$

and it implies that book value of equity stays constant:

$$\begin{aligned}
\underline{\widetilde{V}}^{\mathrm{l}}_{t+1} &= \underline{\widetilde{V}}^{\mathrm{l}}_t + \widetilde{EBIT}_{t+1}(1-\tau) - \widetilde{FCF}^{\mathrm{u}}_{t+1} \\
\underline{\widetilde{V}}^{\mathrm{l}}_{t+1} &= \underline{\widetilde{V}}^{\mathrm{l}}_t - \underline{\widetilde{D}}_t + \underline{\widetilde{D}}_{t+1} \\
\underline{\widetilde{E}}_{t+1} &= \underline{\widetilde{E}}_t .
\end{aligned}$$

The last equation implies: the book value of equity is deterministic. But if the leverage ratio is deterministic as well, debt is deterministic:

$$\underline{\widetilde{D}}_{t+1} = \underline{L}_{t+1}\underline{E}_0$$

Then the adjusted present value approach is appropriate.

Theorem 2.14 (full distribution): *If the firm is financed based on book values and carries out full distribution, then*

$$\widetilde{V}_t^{\mathrm{l}} = \widetilde{V}_t^{\mathrm{u}} + \sum_{s=t+1}^{T} \frac{\tau r_f \underline{L}_{s-1}\underline{E}_0}{(1+r_f)^{s-t}}.$$

Proof: immediate.

Definition 2.11 (replacement investment): *A firm takes on replacement investments if for all t*

$$\widetilde{Inv}_{t+1} = \widetilde{Accr}_{t+1}.$$

Then

$$\widetilde{EBIT}_t(1-\tau) - \widetilde{FCF}^{\mathrm{u}}_t = \widetilde{Inv}_t - \widetilde{Accr}_t = 0.$$

Hence, there is no difference between profits and cash flows of the unlevered firm. Hence, using the operating liabilities relation

$$\underline{\widetilde{V}}^{\mathrm{l}}_{t+1} = \underline{\widetilde{V}}^{\mathrm{l}}_t\ .$$

The last equation implies: the book value of the firm is deterministic. But if the leverage ratio is deterministic as well, debt is deterministic

$$\underline{\widetilde{D}}_{t+1} = \underline{\mathrm{l}}_{t+1} \underline{V}_0^{\mathrm{l}} \; .$$

Then the adjusted present value approach is appropriate.

Theorem 2.15 (replacement investment): *If the firm is financed based on book values and carries out replacement investments, then*

$$\widetilde{V}_t^{\mathrm{l}} = \widetilde{V}_t^{\mathrm{u}} + \sum_{s=t+1}^{T} \frac{\tau r_f \underline{\mathrm{l}}_{s-1} \underline{V}_0^{\mathrm{l}}}{(1+r_f)^{s-t}}$$

Proof: immediate.

Now let us turn to a more realistic case – investments are tied to cash flows:

- increasing cash flows raise investments,
- decreasing cash flows lower investments.

Investments are aligned with the unlevered firm (investments are independent of leverage!), or

$$\widetilde{Inv}_t = \alpha_t \widetilde{FCF}_t^{\mathrm{u}} \qquad \alpha_t > 0.$$

To motivate our assumption let us look at gross cash flows. They are paid to and for

1. tax authority,
2. investments, and
3. shareholder and creditors.

If investments get a fixed portion, it should depend on gross cash flow after taxes

$$\widetilde{Inv}_t = \beta_t \left(\widetilde{GCF}_t - \tau \widetilde{EBIT}_t \right) \qquad 0 < \beta_t < 1.$$

But this implies our assumption since (see (2.21)),

$$\widetilde{Inv}_t = \underbrace{\frac{\beta_t}{1 - \beta_t}}_{=:\alpha_t} \widetilde{FCF}^{\mathrm{u}}_t .$$

Assumption 2.9 (no discretionary accruals): *Accruals follow for all t*

$$\widetilde{Accr}_t = \frac{1}{n}\left(\widetilde{Inv}_{t-1} + \ldots + \widetilde{Inv}_{t-n}\right).$$

It is necessary that past investments Inv_{-1} to Inv_{-n+1} must be known. Our assumption is certainly satisfied if depreciation is the only element of accruals and if we apply straight-line depreciation.

Given all assumptions a valuation theorem can be verified (see Theorem 2.17). Unfortunately, this formula is **very** muddled...

Important drivers for value are:

1. tax rate, riskless interest rate ($\uparrow$, i.e. market value increases with them), cost of capital $k^{E,\mathrm{u}}$ ($\downarrow$, i.e. value decreases with them);
2. today's book value ($\uparrow$), past investments ($\downarrow$);
3. leverage ratio $\underline{\mathrm{l}}$ to book values ($\uparrow$);
4. investment ratio α ($\uparrow$);
5. depreciation period n ($\sim\uparrow$) – requires explanation.

Market value and depreciation

If accruals (depreciation) decrease

$\Longrightarrow$ (given the investment!) book values increase

$\Longrightarrow$ (given the leverage ratio) debt increases

$\Longrightarrow$ tax advantages increase

$\Longrightarrow$ market values increase.

A simpler formula can be obtained for infinite horizon. We assume

- an infinite lifetime,
- a constant debt ratio $\underline{\mathrm{l}}$ and a constant investment parameter α, and
- disregard past investments.

Then (Theorem 2.18)

$$V_0^{\mathrm{l}} \approx V_0^{\mathrm{u}} \left(1 + \frac{n+1}{2} r_f \tau \alpha \underline{\mathrm{l}} \right) + \tau D_0.$$

This formula looks very like Modigliani-Miller, except for the term in brackets!

We use

$$n = 2, \qquad \alpha = 50\%, \qquad \underline{\mathrm{l}} = 50\%.$$

No investments before $t = 0$. Then

$$\begin{aligned} V_0^{\mathrm{l}} &\approx V_0^{\mathrm{u}} + \tau D_0 + \frac{n+1}{2} r_f \tau \alpha \underline{\mathrm{l}} V_0^{\mathrm{u}} \\ &\approx 500 + 0.5 \times 100 + \frac{2+1}{2} 0.1 \times 0.5 \times 0.5 \times 0.5 \times 500 \\ &\approx 559.375. \end{aligned}$$

(*Remark:* Here we use the approximation formula.)

Financing based on book values requires knowledge of the investment policy.

The operating liabilities relation reveals movement of book values.

Three investment policies can be considered:

- full distribution (or debt financed investments) $\Longrightarrow$ APV;
- replacement investments $\Longrightarrow$ APV;
- investment based on cash flows $\Longrightarrow$ MoMi-like formula.