Lecture: Financing Based on Market Values II

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Discounted Cash Flow, Section 2.4.4–2.4.5

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Outline

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Up to now we know three procedures to evaluate $\widetilde{E}_t$ (or $\widetilde{V}_t^{\mathrm{l}}$). In case of financing based on market values these procedures coincide, otherwise not.

But what can we tell about the relation between *WACC* and the unlevered cost of capital, i.e. $k^{E,\mathrm{u}}$?

This is the topic of **adjustment formulas**.

And this time we will need the assumption of weak autoregressive cash flows.

Theorem 2.11 (Miles-Ezzell 1980): *If cash flows of the unlevered firm are weak autoregressive, the levered firm is financed based on market values and WACC is deterministic, then*

$$1 + WACC_t = \left(1 + k_t^{E,\mathrm{u}}\right)\left(1 - \frac{\tau r_f}{1 + r_f}\,\mathrm{l}_t\right)$$

and $k^{E,u}$ is deterministic.

The original article of Miles-Ezzell required a **constant** leverage ratio l_t and riskless debt: **we do not!**

This adjustment formula finally shows why *WACC* might be useful (remember apples and oranges?).

The assumption of weak autoregressive cash flows is necessary (we come back to this).

The proof is not an easy task.

What is the connection to the Modigliani-Miller (1963) adjustment formula? Later. . .

1. write down recursion formula with tax shield in nominator
2. transfer tax shield to denominator (=first part of the proof)
3. *detour*: rewrite recursion formula as infinite sum
4. apply fact that E_Q can be replaced by E if r_f is replaced by cost of capital ("cost of capital are discount rates")
5. rewrite infinite sum as recursion equation (*detour finished*)
6. use definition of WACC

The detour is necessary because our Theorem 2.3 can only be applied to cash flows $\widetilde{FCF}^{\mathrm{u}}$. But in the recursion formulae also $\widetilde{V}^{\mathrm{l}}$ appears!

$$\widetilde{V}_t^{\mathrm{l}} = \frac{\mathsf{E}_Q\left[\widetilde{V}_{t+1}^{\mathrm{l}} + \widetilde{FCF}_{t+1}^{\mathrm{u}} + \tau(\widetilde{I}_{t+1} + \widetilde{R}_{t+1} + \widetilde{D}_{t+1} - \widetilde{D}_t)|\mathcal{F}_t\right]}{1 + r_f}$$

$$\widetilde{V}_t^{\mathrm{l}} = \frac{\mathsf{E}_Q\left[\widetilde{V}_{t+1}^{\mathrm{l}} + \widetilde{FCF}_{t+1}^{\mathrm{u}}|\mathcal{F}_t\right] + \mathsf{E}_Q\left[\tau(\widetilde{I}_{t+1} + \widetilde{R}_{t+1} + \widetilde{D}_{t+1} - \widetilde{D}_t)|\mathcal{F}_t\right]}{1 + r_f}$$

$$\widetilde{V}_t^{\mathrm{l}} = \frac{\mathsf{E}_Q\left[\widetilde{V}_{t+1}^{\mathrm{l}} + \widetilde{FCF}_{t+1}^{\mathrm{u}}|\mathcal{F}_t\right]}{1 + r_f} + \frac{\tau r_f \widetilde{D}_t}{1 + r_f}$$

$$\widetilde{V}_t^{\mathrm{l}} - \frac{\tau r_f \widetilde{D}_t}{1 + r_f} = \frac{\mathsf{E}_Q\left[\widetilde{V}_{t+1}^{\mathrm{l}} + \widetilde{FCF}_{t+1}^{\mathrm{u}}|\mathcal{F}_t\right]}{1 + r_f}$$

$$\widetilde{V}_t^{\mathrm{l}} = \frac{\mathsf{E}_Q\left[\widetilde{V}_{t+1}^{\mathrm{l}} + \widetilde{FCF}_{t+1}^{\mathrm{u}}|\mathcal{F}_t\right]}{\left(1 - \frac{\tau r_f \mathrm{l}_t}{1 + r_f}\right)(1 + r_f)}.$$

Proof (part II) 6

$$\widetilde{V}^{\mathrm{l}}_t = \frac{\mathsf{E}_Q\left[\widetilde{V}^{\mathrm{l}}_{t+1} + \widetilde{FCF}^{\mathrm{u}}_{t+1}|\mathcal{F}_t\right]}{\left(1 - \frac{\tau r_f \mathrm{l}_t}{1+r_f}\right)(1+r_f)}$$

$$\widetilde{V}^{\mathrm{l}}_t = \sum_{s=t+1}^{T} \frac{\mathsf{E}_Q\left[\widetilde{FCF}^{\mathrm{u}}_s|\mathcal{F}_t\right]}{\left(1 - \frac{\tau r_f}{1+r_f}\,\mathrm{l}_{s-1}\right)\cdots\left(1 - \frac{\tau r_f}{1+r_f}\,\mathrm{l}_t\right)(1+r_f)^{s-t}}$$

$$\widetilde{V}^{\mathrm{l}}_t = \sum_{s=t+1}^{T} \frac{\mathsf{E}\left[\widetilde{FCF}^{\mathrm{u}}_s|\mathcal{F}_t\right]}{\left(1 - \frac{\tau r_f}{1+r_f}\,\mathrm{l}_{s-1}\right)\cdots\left(1 - \frac{\tau r_f}{1+r_f}\,\mathrm{l}_t\right)\left(1+k^{E,\mathrm{u}}\right)^{s-t}}$$

$$\widetilde{V}^{\mathrm{l}}_t = \frac{\mathsf{E}\left[\widetilde{V}^{\mathrm{l}}_{t+1} + \widetilde{FCF}^{\mathrm{u}}_{t+1}|\mathcal{F}_t\right]}{\left(1 - \frac{\tau r_f \mathrm{l}_t}{1+r_f}\right)\left(1+k^{E,\mathrm{u}}\right)}$$

$$\left(1 - \frac{\tau r_f l_t}{1+r_f}\right)\left(1+k^{E,\mathrm{u}}\right) = \frac{\mathsf{E}\left[\widetilde{V}^{\mathrm{l}}_{t+1} + \widetilde{FCF}^{\mathrm{u}}_{t+1}|\mathcal{F}_t\right]}{\widetilde{V}^{\mathrm{l}}_t}$$

$$\left(1 - \frac{\tau r_f \mathrm{l}_t}{1+r_f}\right)\left(1+k^{E,\mathrm{u}}\right) = 1 + WACC_t.$$

This equation is the first (1963) adjustment formula. Two assumptions are necessary: constant amount of debt (autonomous financing!) and infinite lifetime. Claims a nice formula:

$$WACC = k^{E,\mathrm{u}}(1-\tau l) \qquad \left(\text{compare to MoMi: } V_0^{\mathrm{l}} = \frac{\mathrm{E}\left[\widetilde{FCF}^{\mathrm{u}}\right]}{(1-\tau \mathrm{l}_0)k^{E,\mathrm{u}}}\right).$$

But this does not seem to relate to the Miles-Ezzell adjustment?!

Theorem 2.12 (Modigliani-Miller): *If the WACC type 2 ($WACC$) and the unlevered firm's cost of equity ($k^{E,u}$) are deterministic, then the firm is financed based on market values.*

This is a **contradiction to autonomous financing**, hence the Modigliani-Miller formula is not applicable!

Summary:

In order to apply MoMi two costs of capital must be deterministic: $WACC$ and $k^{E,\mathrm{u}}$, otherwise the (nice) formula does not make sense.

But then the firm is financed based on market values (see above theorem).

This contradicts the assumption of MoMi (constant debt)!

Hence, there is **no connection to Miles-Ezzell**, since the nice formula is not applicable under any circumstances!

Is the original paper Modigliani-Miller (1963) flawed?

No, since for both (as for Miles-Ezzell as well) costs of capital were not conditional expected returns (but discount rates. . .)!

Although you can use MoMi theorem (Theorem 2.5), you cannot use MoMi adjustment formula.

Put differently: ‘discount rates’ and ‘expected returns’ cover different economic concepts.

Proof 10

$$\left(1-\tau\widetilde{\mathrm{l}}_t\right)\underbrace{\sum_{s=t+1}^{T}\frac{(1+g_t)\dots(1+g_s)\widetilde{FCF}_t^{\mathrm{u}}}{(1+WACC_t)\dots(1+WACC_{s-1})}}_{=\widetilde{V}_t^{\mathrm{l}}}$$

$$=\underbrace{\sum_{s=t+1}^{T}\frac{(1+g_t)\cdots(1+g_s)\widetilde{FCF}_t^{\mathrm{u}}}{\left(1+k_t^{E,\mathrm{u}}\right)\dots\left(1+k_{s-1}^{E,\mathrm{u}}\right)}}_{=\widetilde{V}_t^{u}}.$$

Consider financing based on market values

$$\mathrm{l}_0 = 55\%, \qquad \mathrm{l}_1 = 10\%, \qquad \mathrm{l}_2 = 10\%.$$

WACC results from Miles-Ezzell adjustment (Theorem 2.11),

$$\begin{aligned}
WACC_0 &= \left(1 + k^{E,\mathrm{u}}\right)\left(1 - \frac{\tau r_f}{1 + r_f}\,\mathrm{l}_0\right) - 1 \\
&= (1 + 0.2)\left(1 - \frac{0.5 \times 0.1}{1 + 0.1} \times 0.55\right) - 1 = 17\% \\
WACC_1 &\approx 19.45\% \\
WACC_2 &\approx 19.45\%.
\end{aligned}$$

The firm value is

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\begin{aligned}
V_0^{\mathrm{l}} &= \frac{\mathrm{E}\left[\widetilde{FCF}_1^{\mathrm{u}}\right]}{1+WACC_0} + \frac{\mathrm{E}\left[\widetilde{FCF}_2^{\mathrm{u}}\right]}{(1+WACC_0)(1+WACC_1)} \\
&\quad + \frac{\mathrm{E}\left[\widetilde{FCF}_3^{\mathrm{u}}\right]}{(1+WACC_0)(1+WACC_1)(1+WACC_2)} \\
&\approx \frac{100}{1.17} + \frac{110}{1.17\times 1.1945} + \frac{121}{1.17\times 1.1945\times 1.1945} \approx 236.65.
\end{aligned}
$$

Let $r_f = 10\%$. The leverage ratio is $l = 20\%$ and constant through lifetime.

With Miles-Ezzell adjustment the WACC is

$$WACC = (1 + k^{E,\mathrm{u}}) \left(1 - \frac{\tau r_f}{1 + r_f} l \right) - 1 \approx 18.9091\%$$

and the firm value is

$$\begin{aligned} V_0^{\mathrm{l}} &= \sum_{t=1}^{\infty} \frac{\mathrm{E}\left[\widetilde{FCF}_t^{\mathrm{u}} | \mathcal{F}_0\right]}{(1 + WACC)^t} \\ &= \sum_{t=1}^{\infty} \frac{FCF_0^u}{(1 + WACC)^t} = \frac{FCF_0^u}{WACC} \\ &\approx \frac{100}{0.189091} \approx 528.846. \end{aligned}$$

The connection between costs of capital of a levered firm and an unlevered firm are given by adjustment formulas.

In particular a relation between *WACC* and $k^{E,\mathrm{u}}$ can be proven, hence *WACC* is indeed useful.

The adjustment by Miles-Ezzell is applicable (but this requires financing based on market values!).

The adjustment by Modigliani-Miller is not applicable, although the Modigliani-Miller Theorem can be used.