DCF: Firm income tax
Financing based on market values and WACC

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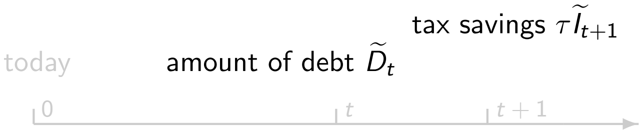
Definition financing based on market values: *Financing is based on market values if* *debt ratios* l_t *are deterministic.*



The time structure of financing based on market values

$\Longrightarrow$ The amount of future debt $\widetilde{D}_t$ is **stochastic**!

$\Longrightarrow$ The tax advantages from debt are **stochastic as well**!

$\Longrightarrow$ APV does not apply! Instead three different procedures. . .

To evaluate the company

1. We start with "appropriate" cost of capital.
2. We assume that these cost of capital is deterministic and apply (as usual) a corresponding valuation formula.
3. We then look at the connection of these procedures: they are given by textbook formulas.

There are three "appropriate" costs of capital, hence there will be three valuation procedures: FTE, TCF, WACC.

Notice that default is not ruled out!

Overview of three procedures:

procedure	**reference value**	**cost of capital**
TCF	$\widetilde{V}$	$\widetilde{k}^{\emptyset}$
FTE	$\widetilde{E}+\widetilde{D}$	$k^{E,l}$
WACC	$\widetilde{V}$	$\widetilde{WACC}$

Every cost of capital is a ratio of corresponding cash flow to the reference value. But there will be an anomaly with WACC...

Now we are looking at the stockholders and the debtholders, or the cost of equity and debt.

Definition weighted average cost of capital—type 1: *WACC type 1 is a conditional expected return*

$$\widetilde{k}_t^{\emptyset} := \frac{\mathrm{E}_t\left[\widetilde{CF}^{l}_{t+1} + \widetilde{V}^{l}_{t+1}\right]}{\widetilde{V}^{l}_t} - 1 \,.$$

Theorem TCF: *If* $\widetilde{k}_t^{\emptyset}$ *is deterministic, then*

$$\widetilde{V}_t^{l} = \sum_{s=t+1}^{T} \frac{\mathrm{E}_t\left[\widetilde{CF}_s^{l}\right]}{\left(1+k_t^{\emptyset}\right)\cdots\left(1+k_{s-1}^{\emptyset}\right)} \,.$$

Proof: see our general valuation theorem.
Remarks:

- TCF requires deterministic WACC type 1.
- The theorem does not yet require market-value-based financing.
- The leverage ratio does not appear in TCF.
- The knowledge of expected debt is not necessary.

With FTE we are looking at the stockholders and their *cost of equity*. The cash flow to stockholders is given by

	free cash flows	$\widetilde{CF}^{\mathrm{l}}_{t+1}$
$-$	interest and principal	$-\widetilde{I}_{t+1} - \widetilde{Pr}_{t+1}.$

Definition cost of equity: *Costs of equity is the conditional expected return*[1]

$$\widetilde{k}^{E,\mathrm{l}}_t := \frac{\mathrm{E}_t\left[\widetilde{E}_{t+1} + \widetilde{CF}^{\mathrm{l}}_{t+1} - \widetilde{I}_{t+1} - \widetilde{Pr}_{t+1}\right]}{\widetilde{E}_t} - 1\,.$$

[1] Keep the expectation E_t and the market value of equity $\widetilde{E}_t$ distinct: the expectation carries no tilde, whereas equity does.

Theorem FTE: *If $\widetilde{k}_t^{E,l}$ is deterministic, then*

$$\widetilde{E}_t = \sum_{s=t+1}^{T} \frac{\mathrm{E}_t\left[\widetilde{CF}_s^{\,l} - \widetilde{I}_s - \widetilde{Pr}_s\right]}{\left(1+k_t^{E,l}\right)\cdots\left(1+k_{s-1}^{E,l}\right)}.$$

Proof: see our general valuation theorem.

Remarks:

- FTE requires deterministic cost of equity.
- The theorem does not yet require financing based on market values!
- The leverage ratio does not appear in FTE.
- The knowledge of expected repayment is necessary.

What is the connection between FTE and TCF? The answer is the textbook formula.

Theorem TCF textbook formula: *It always holds that*

$$\widetilde{k}_t^{\emptyset} = \widetilde{k}_t^{E,\mathrm{l}} \left(1 - \widetilde{\mathrm{l}}_t\right) + \widetilde{k}_t^{D} \widetilde{\mathrm{l}}_t \ .$$

$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t = \mathrm{E}_t\left[\widetilde{E}_{t+1}+\widetilde{CF}^{\mathrm{l}}_{t+1}-\widetilde{Pr}_{t+1}+\widetilde{D}_{t+1}-\widetilde{D}_{t+1}-\widetilde{I}_{t+1}\right]$$

$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t = \mathrm{E}_t\left[\widetilde{V}^{\mathrm{l}}_{t+1}+\widetilde{CF}^{\mathrm{l}}_{t+1}-\widetilde{Pr}_{t+1}-\widetilde{D}_{t+1}-\widetilde{I}_{t+1}\right]$$

$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t = \mathrm{E}_t\left[\widetilde{V}^{\mathrm{l}}_{t+1}+\widetilde{CF}^{\mathrm{l}}_{t+1}-\widetilde{D}_t-\widetilde{k}_t^D\,\widetilde{D}_t\right]$$

$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t+\left(1+\widetilde{k}_t^D\right)\widetilde{D}_t = \mathrm{E}_t\left[\widetilde{V}^{\mathrm{l}}_{t+1}+\widetilde{CF}^{\mathrm{l}}_{t+1}\right]$$

$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t+\left(1+\widetilde{k}_t^D\right)\widetilde{D}_t = \left(1+\widetilde{k}_t^{\emptyset}\right)\widetilde{V}^{\mathrm{l}}_t$$

$$\widetilde{E}_t+\widetilde{k}_t^{E,\mathrm{l}}\widetilde{E}_t+\widetilde{D}_t+\widetilde{k}_t^D\widetilde{D}_t = \widetilde{V}^{\mathrm{l}}_t+\widetilde{k}_t^{\emptyset}\widetilde{V}^{\mathrm{l}}_t$$

$$\widetilde{k}_t^{E,\mathrm{l}}\,\frac{\widetilde{E}_t}{\widetilde{V}^{\mathrm{l}}_t}+\widetilde{k}_t^D\,\frac{\widetilde{D}_t}{\widetilde{V}^{\mathrm{l}}_t} = \widetilde{k}_t^{\emptyset}.$$

Remarks to TCF, I

In the TCF textbook formula, the cost of debt is not reduced by the tax rate. The formula holds regardless of whether the relevant variables are deterministic or stochastic.

In particular: financing based on market values is not necessary!

Assume no default. One of two cases possible

market-value financing If WACC type 1 or cost of equity is deterministic, the other is deterministic as well. TCF and FTE can be used simultaneously.

non market-value financing Either WACC type 1 or cost of equity has to be uncertain. TCF and FTE exclude each other.

Proof:

$$\widetilde{k}_t^{\emptyset} = \widetilde{k}_t^{E,\mathrm{l}} \left(1 - \widetilde{\mathrm{l}}_t\right) + r_f \, \widetilde{\mathrm{l}}_t \ .$$

We are back again to stockholders and debtholders.

Definition weighted average cost of capital—type 2: *WACC type 2 is the conditional expected return*

$$\widetilde{WACC}_t := \frac{\mathrm{E}_t\left[\widetilde{V}^{\mathrm{l}}_{t+1} + \widetilde{CF}^{\mathrm{u}}_{t+1}\right]}{\widetilde{V}^{\mathrm{l}}_t} - 1 \,.$$

Remark: This is a "cost of capital" of a firm that is on the one hand levered ($\widetilde{V}^{\mathbf{l}}_t$) and on the other hand unlevered ($\widetilde{CF}^{\mathbf{u}}_{t+1}$). Apples and oranges mixed here.

Theorem WACC: *If* $\widetilde{WACC}_t$ *is deterministic, then*

$$\widetilde{V}_t^{\mathrm{l}} = \sum_{s=t+1}^{T} \frac{\mathrm{E}_t\left[\widetilde{CF}_s^{\mathrm{u}}\right]}{(1+WACC_t)\cdots(1+WACC_{s-1})}.$$

Proof: see our general valuation theorem

Remarks:

- WACC requires deterministic WACC—type 2.
- The theorem above does not yet require financing based on market values!
- The leverage ratio does not appear in WACC.
- The knowledge of cash flow of an unlevered firm is necessary.

WACC textbook formula

What is the connection between FTE and WACC? The answer is another textbook formula.

Theorem WACC textbook formula: *Always*

$$\widetilde{WACC}_t = \widetilde{k}_t^{E,\mathrm{l}} \left(1 - \widetilde{\mathrm{l}}_t\right) + \widetilde{k}_t^D (1 - \tau) \widetilde{\mathrm{l}}_t.$$

$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t = \mathrm{E}_t\left[\widetilde{E}_{t+1}+\widetilde{CF}^{\mathrm{l}}_{t+1}-\widetilde{Pr}_{t+1}-\widetilde{I}_{t+1}\right]$$
$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t = \mathrm{E}_t\left[\widetilde{V}^{\mathrm{l}}_{t+1}+\widetilde{CF}^{\mathrm{l}}_{t+1}-\widetilde{Pr}_{t+1}-\widetilde{I}_{t+1}-\widetilde{D}_{t+1}\right]$$
$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t = \mathrm{E}_t\left[\widetilde{V}^{\mathrm{l}}_{t+1}+\widetilde{CF}^{\mathrm{u}}_{t+1}-\widetilde{Pr}_{t+1}-\widetilde{I}_{t+1}-\widetilde{D}_{t+1}+\tau(\widetilde{I}_{t+1}-\widetilde{D}_t+\widetilde{Pr}_{t+1}+\widetilde{D}_{t+1})\right]$$
$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t = \mathrm{E}_t\left[\widetilde{V}^{\mathrm{l}}_{t+1}+\widetilde{CF}^{\mathrm{u}}_{t+1}-(1+\widetilde{k}^D)\widetilde{D}_t+\tau\,\widetilde{k}^D\widetilde{D}_t\right].$$

Proof, continued

$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t+\left(1+\widetilde{k}^D(1-\tau)\right)\widetilde{D}_t=\mathsf{E}_t\left[\widetilde{V}_{t+1}^{\mathrm{l}}+\widetilde{CF}_{t+1}^{\mathrm{u}}\right]$$

$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t+\left(1+\widetilde{k}^D(1-\tau)\right)\widetilde{D}_t=(1+WACC_t)\,\widetilde{V}_t^{\mathrm{l}}$$

$$\widetilde{E}_t+\widetilde{k}_t^{E,\mathrm{l}}\widetilde{E}_t+\widetilde{D}_t+\widetilde{k}^D(1-\tau)\widetilde{D}_t=\widetilde{V}_t^{\mathrm{l}}+WACC_t\,\widetilde{V}_t^{\mathrm{l}}$$

$$\widetilde{k}_t^{E,\mathrm{l}}\,\frac{\widetilde{E}_t}{\widetilde{V}_t^{\mathrm{l}}}+\widetilde{k}^D(1-\tau)\,\frac{\widetilde{D}_t}{\widetilde{V}_t^{\mathrm{l}}}=WACC_t.$$

And this was to be shown QED

The cost of debt is reduced by the tax rate in the WACC textbook formula. The formula holds regardless of whether the relevant variables are deterministic or stochastic.

In particular: financing based on market values is not necessary!

Assume no default. One of two cases is possible.

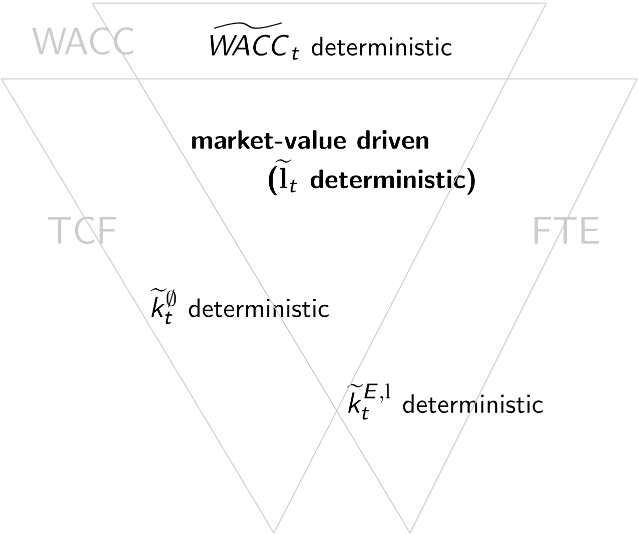
market-value financing If WACC type 2 or cost of equity is deterministic, the other is deterministic as well. WACC and FTE can be used simultaneously.

non market-value financing Either WACC type 2 or cost of equity has to be uncertain. WACC and FTE exclude each other.

Proof:

$$\widetilde{WACC}_t = \widetilde{k}_t^{E,l} \left(1 - \widetilde{l}_t\right) + r_f \left(1 - \tau\right) \widetilde{l}_t .$$

Summary


WACC
$\widetilde{WACC}_t$ deterministic
market-value driven
($\widetilde{l}_t$ deterministic)
TCF
FTE
$\widetilde{k}_t^{\emptyset}$ deterministic
$\widetilde{k}_t^{E,l}$ deterministic