

DCF: Firm income tax
Autonomous financing and adjusted present value
(APV)

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Definition autonomous financing: *A firm is autonomously financed if its future debt level \tilde{D}_t is deterministic.*

Theorem APV: *If a firm is autonomously financed, then*

$$\tilde{V}_t^l = \tilde{V}_t^u + \sum_{s=t+1}^T \frac{\tau r_f D_{s-1}}{(1+r_f)^{s-t}}.$$

Proof: trivial.



Theorem constant debt, MoMi: *If $T = \infty$ and $D_t = \text{const}$, then*

$$\tilde{V}_t^l = \tilde{V}_t^u + \tau D_t.$$

Proof:

$$\begin{aligned}\tilde{V}_t^l &= \tilde{V}_t^u + \sum_{s=t+1}^T \frac{\tau r_f D_{s-1}}{(1+r_f)^{s-t}} \\ &= \tilde{V}_t^u + \tau r_f D_t \sum_{s=t+1}^T \frac{1}{(1+r_f)^{s-t}} \\ &= \tilde{V}_t^u + \tau r_f D_t \frac{1}{r_f}.\end{aligned}$$



A different formula would be

$$\begin{aligned}V_0^l &= V_0^u + \tau D_0 \\V_0^l &= V_0^u + \tau l_0 V_0^l \\(1 - \tau l_0) V_0^l &= V_0^u \\V_0^l &= \frac{E[\widetilde{CF}^u]}{(1 - \tau l_0) k^{E,u}}.\end{aligned}$$

We come back to this (or at least a similar) equation in the next lecture when talking about the **Modigliani-Miller adjustment**.



We now look at our finite example with $\tau = 50\%$.

$$D_0 = 100, \quad D_1 = 100, \quad D_2 = 50.$$

Hence,

$$\begin{aligned} V_0^l &= V_0^u + \frac{\tau r_f D_0}{1 + r_f} + \frac{\tau r_f D_1}{(1 + r_f)^2} + \frac{\tau r_f D_2}{(1 + r_f)^3} \\ &\approx 229.75 + \frac{0.5 \times 0.1 \times 100}{1 + 0.1} + \frac{0.5 \times 0.1 \times 100}{(1 + 0.1)^2} \\ &\quad + \frac{0.5 \times 0.1 \times 50}{(1 + 0.1)^3} \approx 240.30. \end{aligned}$$

This is also the value of the firm threatened by default.



For later use we evaluate the future firm value \tilde{V}_1^1 ,

$$\begin{aligned}\tilde{V}_1^1 &= \tilde{V}_1^u + \frac{\tau r_f D_1}{1 + r_f} + \frac{\tau r_f D_2}{(1 + r_f)^2} \\ &= \tilde{V}_1^u + \frac{0.5 \times 0.1 \times 100}{1 + 0.1} + \frac{0.5 \times 0.1 \times 50}{(1 + 0.1)^2} \\ &\approx \begin{cases} 199.88 & \text{if up,} \\ 164.74 & \text{if down.} \end{cases}\end{aligned}$$



Be careful! The corresponding debt ratio

$$\tilde{l}_1 = \frac{D_1}{\tilde{V}_1^1} \approx \begin{cases} 50.03 \% & \text{if up,} \\ 60.03 \% & \text{if down,} \end{cases}$$

is **uncertain!**

Hence, a deterministic debt level implies a stochastic leverage ratio—and vice versa. . .



Since we can still use the APV-formula,

$$\tilde{V}_0^1 \approx 240.30.$$

Another way of obtaining this value is by evaluating $E_0^Q[\tilde{CF}_t^1]$ and discounting it with the risk-free rate.



Assume $\tau = 50\%$ and constant debt

$$D_t = 100.$$

Then

$$\begin{aligned} V_0^l &= V_0^u + \tau D_0 \\ &= 550. \end{aligned}$$



With autonomous financing the amount of debt is constant.

The valuation of the tax shield is straightforward.

The leverage ratio is in this case random.

