

DCF: Firm income tax Insolvency and Valuation

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In this lecture we are interested in the valuation aspects of insolvency and will not yet specify an insolvency trigger (illiquidity, negative equity, . . .).

Without insolvency valuation equation was

$$\tilde{V}_t^l = \tilde{V}_t^u + \frac{\tau r_f E_t^Q [\tilde{D}_t]}{1 + r_f} + \dots + \frac{\tau r_f E_t^Q [\tilde{D}_{T-1}]}{(1 + r_f)^{T-t}}$$

How does insolvency alter the valuation implied by this equation?



Owners have no personal liability.

Creditors and shareholders have identical information about the company, its estate, the firm's cost of capital and its financing policy.

Identical gross cash flows: *The gross cash flows (i.e., before debt payments) as well as the investment and accruals policy of the unlevered firm do not differ from those in case of insolvency.*



Who gets what in case of insolvency?

Prioritization of debt: *The tax office's claims range before those of other creditors. The cash flows are always sufficient to at least pay off the tax debts in full.*

Now a new notation is necessary:

\tilde{D}_t amount of credit outstanding in t
 \tilde{Pr}_{t+1} principal that is paid back in $t + 1$
 \tilde{I}_{t+1} interest paid in $t + 1$

How is interest determined? We will see later (“coupon c ”).



The tax authority permits interest \tilde{I}_{t+1} to be deducted from the tax base. By contrast, the cancellation of debt

$$\underbrace{\tilde{D}_t - \tilde{D}_{t+1}}_{\text{should be paid back}} - \underbrace{\tilde{Pr}_{t+1}}_{\text{was paid back}}$$

adds to the tax base ('recapitalization gain'). Hence, the **tax due** in the event of insolvency is

$$\widetilde{Tax}_{t+1}^1 = \tau \left(\widetilde{GCF}_{t+1} - \widetilde{Accr}_{t+1} - \underbrace{\tilde{I}_{t+1}}_{\substack{\text{interest,} \\ \text{deducted}}} + \underbrace{\tilde{D}_t - \tilde{D}_{t+1} - \tilde{Pr}_{t+1}}_{\substack{\text{gain,} \\ \text{added}}} \right).$$



By comparing unlevered and levered cash flows, we obtain

$$\begin{aligned}\widetilde{CF}_{t+1}^l &= \widetilde{CF}_{t+1}^u + \widetilde{Tax}_{t+1}^u - \widetilde{Tax}_{t+1}^l \\ &= \widetilde{CF}_{t+1}^u + \tau \left(\widetilde{I}_{t+1} + \widetilde{Pr}_{t+1} + \widetilde{D}_{t+1} - \widetilde{D}_t \right)\end{aligned}$$

and the **main valuation equation** now reads

$$\widetilde{V}_t^l = \widetilde{V}_t^u + \sum_{s=t+1}^T \frac{\tau E_t^Q \left[\widetilde{I}_s + \widetilde{Pr}_s + \widetilde{D}_s - \widetilde{D}_{s-1} \right]}{(1+r_f)^{s-t}}. \quad (**)$$

This does not look like the equation without insolvency?! But let us see. . .



The **debtholders behave rational**. Therefore, at time s

$$\tilde{D}_{s-1} = \frac{E_{s-1}^Q [\text{value in } s]}{1 + r_f} = \frac{E_{s-1}^Q [\tilde{P}r_s + \tilde{D}_s + \tilde{l}_s]}{1 + r_f}$$

and from rule 2 and 5

$$r_f E_{s-1}^Q [\tilde{D}_{s-1}] = E_{s-1}^Q [\tilde{l}_s + \tilde{P}r_s + \tilde{D}_s - \tilde{D}_{s-1}].$$

and from rule 4 for all $t \leq s$

$$r_f E_t^Q [\tilde{D}_{s-1}] = E_t^Q [\tilde{l}_s + \tilde{P}r_s + \tilde{D}_s - \tilde{D}_{s-1}].$$

And the rhs is the nominator from the main valuation equation without insolvency!



Even in the case of insolvency the earlier (“basic”) valuation equation

$$\tilde{V}_t^l = \tilde{V}_t^u + \frac{\tau r_f E_t^Q [\tilde{D}_t]}{1 + r_f} + \dots + \frac{\tau r_f E_t^Q [\tilde{D}_{T-1}]}{(1 + r_f)^{T-t}}$$

holds.

Or: If debt is issued, insolvency can be ignored when valuing. Insolvency per se has no influence on the value.



What is the intuition of our result?

Owners and creditors have the **same information about the firm**. Hence, when debt is issued, creditors understand that insolvency is possible and price the debt accordingly.

(Put differently, for insolvency risk to affect value, it must alter either the future cash flows or the applicable tax bases. And under the point-in-time principle, this effect must already be reflected today.)



Someone who invests \tilde{D}_t today is entitled to payments amounting to $\tilde{D}_t + \tilde{I}_{t+1}$ less remission of debts $\tilde{D}_t - \tilde{D}_{t+1} - \tilde{P}r_{t+1}$. Hence,

Cost of debt: *The cost of debt of a levered firm is*

$$\tilde{k}_t^D = \frac{E_t[\tilde{D}_{t+1} + \tilde{I}_{t+1} + \tilde{P}r_{t+1}]}{\tilde{D}_t} - 1.$$

If there is no insolvency $\tilde{k}_t^D = r_f$.

We do not require the cost of debt to be deterministic today. Cost of debt will not be used itself to determine the value of firms.



DCF remains valid even in the presence of insolvency, because creditors and owners have the same information.

Only tax effects drive changes in firm value in insolvency.

